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Professor R. A. Sampson, M.A., D.Sc., F.R.S., PRESIDENT, in the Chair.

George Ridsdale Goldsbrough, M.Sc., 14 Athol Park, Sunderland,

was balloted for and duly elected a Fellow of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Park Goff, M.A., B.C.L., F.R.G.S., Barrister, Inner Temple, Carlton Club, Pall Mall, S.W. (proposed by Count de Miremont);

François Henroteau, D. ès Sc., Stonyhurst College Observatory, Blackburn (proposed by the Rev. A. L. Cortie);

Miss M. T. Meyer, Director of Studies in Mathematics, Girton College, Cambridge (proposed by J. W. Nicholson);

Lieut. William Robert Patterson, D.A.C., Censor's Office, B.E. Force, Quai Maritime, Boulogne, and 241 Kingston Road, Merton Park, Wimbledon, S.W. (proposed by H. B. Adames);

Miss Mary Proctor, Lecturer in Astronomy, 5 Brunswick Square, W.C. (proposed by R. A. Sampson).

Forty-two presents were announced as having been received since the last meeting, including, amongst others:—

Tycho Brahe, Opera Omnia, edidit J. L. E. Dreyer, Tomus II.,

presented by Dr. Dreyer; Stereoscopic photograph of the H_α flocculi surrounding a large spot group, taken at the Mount Wilson Solar Observatory, 1915 August 7, presented by Professor Hale; Photographs of Jupiter taken with the 28-inch reflector, 1915 November, presented by Mr. J. H. Reynolds.

On the Theory of Star-Streaming and the Structure of the Universe. By J. H. Jeans, M.A., F.R.S.

The present paper attempts to answer some questions suggested by reading recent papers of Professors Eddington and Turner. The main conclusion reached is that star-streaming is evidence that our universe has not yet reached a final steady state. It is not, therefore, possible to derive any evidence as to the structure of the universe by combining our knowledge of star-streaming with the assumption that the universe is in a steady state.

The first three sections of the paper deal with three distinct problems of a rather mathematical nature: a summary and discussion of results will be found in the fourth section.

I. *On Schwarzschild's Ellipsoidal Law.*

Let $f(u, v, w, x, y, z)$ denote any law of distribution of stellar velocities so that the number of stars lying within a small region $dxdydz$ in space, and having components of velocity lying within a small range of $dudvdw$, will be

$$f(u, v, w, x, y, z)dudvdwdxdydz \quad . \quad . \quad . \quad (1)$$

Let X, Y, Z be the components of gravitational force per unit mass at the point x, y, z. Then after a short interval of time dt the group of stars enumerated in formula (1) will have velocity components lying within a small range $dudvdw$ surrounding the values

$$u + Xdt, \quad v + Ydt, \quad w + Zdt,$$

and will similarly have positional co-ordinates within a range $dxdydz$ surrounding the point

$$x + udt, \quad y + vdt, \quad z + wdt.$$

The number of stars having values of u, v, w, x, y, z lying within these limits is, as in formula (1),

$$f(u + Xdt, v + Ydt, w + Zdt, x + udt, y + vdt, z + wdt)dudvdwdxdydz \quad (2)$$

If this expression is evaluated at time $t + dt$, the number of stars enumerated in it must be identical with that enumerated in expression (1) evaluated at time t, for in fact the two groups of

stars are identical. Expanding formula (2) as far as first powers of dt, and equating to expression (1), we obtain at once

$$-\frac{df}{dt} = X\frac{\partial f}{\partial u} + Y\frac{\partial f}{\partial v} + Z\frac{\partial f}{\partial w} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial z} \quad . \quad . \quad (3)$$

The condition that any law of distribution f shall be permanent is that the right-hand member of this equation shall vanish identically for all values of x, y, z, u, v, w. This equation is, of course, the astronomical analogue of Boltzmann's well-known characteristic equation in the kinetic theory of gases.*

The most general law of distribution possible can be expressed in the form

$$f = Ae^{-\phi} = e^{\log A - \phi} \quad . \quad . \quad . \quad . \quad (4)$$

in which A depends only on x, y, and z, while ϕ depends on x, y, z and also on u, v, w, but contains no terms which are independent of u, v, and w. Let us further put

$$\phi = \phi_1 + \phi_2 + \phi_3 + \ . \ . \ .$$

where $\phi_1, \phi_2, \phi_3, \ . \ . \ .$ are terms of degrees 1, 2, 3, . . . respectively in u, v, w.

Putting $df/dt = 0$ in equation (3) and dividing through by f we obtain, as the condition that f shall be a possible law of distribution in a steady state,

$$\left(X\frac{\partial}{\partial u} + Y\frac{\partial}{\partial v} + Z\frac{\partial}{\partial w} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(\phi_1 + \phi_2 + \phi_3 + \ . \ . \ .\right) = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\log A \quad (5)$$

The right-hand member of this equation is of unit degree in u, v, w, and so can clearly be balanced by the terms on the left arising from $\phi_2, \phi_4, \phi_6 . \ . \ .$ The terms of degrees 1, 3, 5 . . . in ϕ may either be absent altogether or may form a separate and independent solution.

We shall at present confine our attention to the solution involving $\phi_2, \phi_4, \phi_6 . \ . \ .$ Equating terms of corresponding degree in equation (5) we obtain

$$X\frac{\partial\phi_2}{\partial u} + Y\frac{\partial\phi_2}{\partial v} + Z\frac{\partial\phi_2}{\partial w} = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\log A \quad . \quad (6)$$

$$X\frac{\partial\phi_4}{\partial u} + Y\frac{\partial\phi_4}{\partial v} + Z\frac{\partial\phi_4}{\partial w} = -\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\phi_2 \quad . \quad (7)$$

and an infinite series of equations similar to (7), except that ϕ_4, ϕ_2 are replaced by ϕ_{4+2n}, ϕ_{2+2n} respectively, where n is any integer from 1 to ∞.

* Boltzmann, *Vorlesungen über Gastheorie*, i. p. 122; or Jeans, *Dynamical Theory of Gases* (2nd edition), equation (552).

Assuming for ϕ_2 the most general value possible, say

$$\phi_2 = au^2 + bv^2 + cw^2 + 2fvw + 2gwu + 2huv \quad . \quad . \quad (8)$$

where a, b, c, f, g, h are functions of x, y, z, and inserting this value for ϕ_2 into equation (6), we obtain

$$\left.\begin{aligned} aX + hY + gZ &= \frac{1}{2A}\frac{\partial A}{\partial x} \\ hX + bY + fZ &= \frac{1}{2A}\frac{\partial A}{\partial y} \\ gX + fY + cZ &= \frac{1}{2A}\frac{\partial A}{\partial z} \end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

Proceeding in this way, we might investigate the most general solution possible for ϕ. Let us, however, confine ourselves to examining solutions which are of Schwarzschild's ellipsoidal form.

For the ellipsoidal law of distribution of velocities to prevail, $\phi_4, \phi_6, \ldots$ must all vanish. Thus equation (7) reduces to

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(au^2 + bv^2 + cw^2 + 2fvw + 2gwu + 2huv\right) = 0,$$

and on equating coefficients of different terms in u, v, w we obtain

$$\frac{\partial a}{\partial x} = \frac{\partial b}{\partial y} = \frac{\partial c}{\partial z} = 0 \quad . \quad . \quad . \quad . \quad (10)$$

$$\frac{\partial a}{\partial y} = -2\frac{\partial h}{\partial x}; \quad \frac{\partial b}{\partial x} = -2\frac{\partial h}{\partial y}; \text{ etc.} \quad . \quad . \quad (11)$$

$$\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} = 0 \quad . \quad . \quad . \quad . \quad (12)$$

Here are ten equations which must be satisfied at all points of space. From equations (10) and (11) we obtain

$$\frac{\partial^2 h}{\partial x^2} = -\frac{1}{2}\frac{\partial^2 a}{\partial x \partial y} = 0,$$

and there are, of course, five other similar equations. It follows that f, g, h must be of the forms

$$\left.\begin{aligned} f &= f_0 + yX_1 + zX_2 + yzX_3 \\ g &= g_0 + zY_1 + xY_2 + zxY_3 \\ h &= h_0 + xZ_1 + yZ_2 + xyZ_3 \end{aligned}\right\} \quad . \quad . \quad . \quad (13)$$

where f_0, g_0, h_0 are constants, X_1, X_2, X_3 are functions of x only, Y_1, Y_2, Y_3 are functions of y only, and so on.

From equations (11),

$$\frac{\partial^2 h}{\partial x \partial z} = -\frac{1}{2}\frac{\partial^2 a}{\partial y \partial z} = \frac{\partial^2 g}{\partial x \partial y}.$$

On substituting the values (13) for h and g, this becomes

$$Z_1' + yZ_3' = Y_2' + zY_3'$$

where Z_1' is written for $\partial Z_1/\partial z_1$, and so on. In order that this may be satisfied at every point in space, we must have

$$Z_1' = Y_2' = \text{a constant} \quad . \quad . \quad . \quad (14)$$

$$yZ_3' = zY_3' \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and there are, of course, two other similar sets of equations.

From equation (15) and its two symmetrical companions we have

$$\frac{1}{y}\frac{\partial Y_3}{\partial y} = \frac{1}{z}\frac{\partial Z_3}{\partial z} = \frac{1}{x}\frac{\partial X_3}{\partial x} = \delta, \text{ say,}$$

where δ is a constant independent of x, y, and z. Thus $X_3' = x\delta$, etc., and equation (12) now becomes

$$x(Z_1' + Y_2') + y(X_1' + Z_2') + z(Y_1' + X_2') + 3\delta xyz = 0.$$

This must be satisfied at all points of space. On combining it with equation (14) and its companion equations we find that it can only be satisfied if

$$Z_1' = Y_2' = X_1' = Z_2' = Y_1' = X_2' = 0,$$

and also if $\delta = 0$, from which it at once follows that

$$X_3' = Y_3' = Z_3' = 0.$$

Thus all the coefficients in equations (13) must reduce to constants.

From relations (11),

$$\frac{\partial a}{\partial y} = -2\frac{\partial h}{\partial x} = -2(Z_1 + yZ_3),$$

$$\frac{\partial a}{\partial x} = -2\frac{\partial g}{\partial x} = -2(Y_2 + zY_3),$$

whence, by simple integration of this and similar equations,

$$\left.\begin{aligned} a &= a_0 - 2(yZ_1 + zY_2) - (y^2Z_3 + z^2Y_3) \\ b &= b_0 - 2(zX_1 + xZ_2) - (z^2X_3 + x^2Z_3) \\ c &= c_0 - 2(xY_1 + yX_2) - (x^2Y_3 + y^2X_3) \end{aligned}\right\} \quad . \quad . \quad (16)$$

From the values of a, b, c, f, g, h which have now been found we obtain, on collecting terms,

$$\begin{aligned}\phi_2 = {} & a_0u^2 + b_0v^2 + c_0w^2 + 2f_0vw + 2g_0wu + 2h_0uv \\ & + 2\Sigma(wX_2 - vX_1)(zv - yw) - \Sigma X_3(zv - yw)^2 \quad (17)\end{aligned}$$

In this value for ϕ_2 there are fifteen constants. These are so far undetermined, but they are not at our disposal, since they have to be adjusted so that equations (9) shall be satisfied.

Let Ω be the gravitational potential at x, y, z, so that $X = \partial\Omega/\partial x$, etc. Then equations (9) become

$$\left(a\frac{\partial}{\partial x}+h\frac{\partial}{\partial y}+g\frac{\partial}{\partial z}\right)\Omega=\frac{1}{2}\frac{\partial}{\partial x}\log A \quad . \quad . \quad (18)$$

$$\left(h\frac{\partial}{\partial x}+b\frac{\partial}{\partial y}+f\frac{\partial}{\partial z}\right)\Omega=\frac{1}{2}\frac{\partial}{\partial y}\log A \quad . \quad . \quad (19)$$

$$\left(g\frac{\partial}{\partial x}+f\frac{\partial}{\partial y}+c\frac{\partial}{\partial z}\right)\Omega=\frac{1}{2}\frac{\partial}{\partial z}\log A \quad . \quad . \quad (20)$$

By simple differentiation we eliminate A, and obtain

$$\frac{\partial}{\partial z}\left(h\frac{\partial}{\partial x}+b\frac{\partial}{\partial y}+f\frac{\partial}{\partial z}\right)\Omega=\frac{\partial}{\partial y}\left(g\frac{\partial}{\partial x}+f\frac{\partial}{\partial y}+c\frac{\partial}{\partial z}\right)\Omega \quad . \quad (21)$$

and two similar equations. On rearranging the terms and simplifying from equations (11) and (13), this equation becomes

$$(b-c)\frac{\partial^2\Omega}{\partial y\partial z}+h\frac{\partial^2\Omega}{\partial x\partial z}-g\frac{\partial^2\Omega}{\partial x\partial y}+f\left(\frac{\partial^2\Omega}{\partial z^2}-\frac{\partial^2\Omega}{\partial y^2}\right)+3\left(\frac{\partial f}{\partial z}\frac{\partial\Omega}{\partial z}-\frac{\partial f}{\partial y}\frac{\partial\Omega}{\partial y}\right)=0 \quad (22)$$

Let us now assume a general value for Ω, say,

$$\begin{aligned}\Omega = {} & c_0+c_1x+c_2y+c_3z \\ & +c_{11}x^2+c_{22}y^2+c_{33}z^2+c_{12}xy+c_{23}yz+c_{31}zx \\ & +c_{111}x^3+c_{222}y^3+\ldots\end{aligned}$$

Then equation (22) becomes

$$\begin{aligned}&(b-c)[c_{23}+c_{231}x+2c_{223}y+2c_{233}z+\ldots \\ &+h[c_{13}+2c_{113}x+c_{123}y+2c_{133}z+\ldots \\ &-g[c_{12}+2c_{112}x+2c_{122}y+c_{123}z+\ldots \\ &+f[2(c_{33}-c_{22})+2(c_{331}-c_{221})x+2(c_{332}-3c_{222})y+\ldots \\ &+3\{(X_2+yX_3)(c_3+c_{13}x+\ldots) \\ &\qquad -(X_1+zX_3)(c_2+c_{21}x+\ldots)\} \quad (23)\end{aligned}$$

and there will, of course, be two other similar equations obtained from this one by a cyclic interchange of letters.

The ellipsoidal law of distribution can only be consistent with steady motion if equation (23) and the two similar equations are satisfied for all values of x, y, z. Equating coefficients of the different products and powers of x, y, z separately to zero, we obtain an infinite series of equations to be satisfied, and there are only fifteen adjustable constants at our disposal with which to satisfy these equations.

It is at once obvious that, for any value of Ω whatever, equation (23) can always be satisfied by taking

$$b-c=f=g=h=0,$$

so that the system of three equations can always be satisfied by taking

$$f = g = h = 0\,;\ a = b = c = q, \text{ a constant} \quad . \quad . \quad (24)$$

But in this solution the ellipsoid in the law of distribution reduces to the sphere $q(u^2 + v^2 + w^2)$. Thus at every point of the universe stellar motions occur equally in all directions and there can be no star-streaming anywhere.

The complete solution resulting from equations (24) is readily obtained. For equations (18)–(20) at once reduce to

$$q\frac{\partial \Omega}{\partial x} = \frac{1}{2}\frac{\partial}{\partial x}(\log A)$$

and two similar equations. All of these are satisfied by taking

$$\log A = 2q(\Omega - B)$$

where B is a constant, and the law of distribution (4) becomes

$$f = e^{\log A - \phi_2} = Ce^{-q(u^2+v^2+w^2-2\Omega)} \quad . \quad . \quad . \quad (25)$$

where C is a new constant.

As we have seen, this is a solution in which star-streaming is impossible. Our problem is rather to search whether there are other solutions for which star-streaming is possible. We must therefore try to satisfy equations (23) in some other way than by the vanishing of the multipliers b–c, f, g, h, etc., and this can only be by the vanishing of the coefficients of x, y, z, x^2, y^2, . . . etc. separately in each equation.

If the arrangement of the universe is thought of as known, the coefficients c_1, c_2, c_3, c_{11}, . . . must all be thought of as being already determined, and it is at once clear that by assigning special values to the limited number (15) of constants at our disposal we cannot hope to satisfy the infinite series of equations except in very special cases. It is, however, precisely these special cases that we are in search for. It has now been seen that, consistently with steady motion, star-streaming can only be possible in very special types of universes, and we must try to find what these special types are. We must, in fact, adjust our constants c_{11}, c_{12}, . . . so that equations (23), etc., may be capable of being satisfied by the adjustment of the fifteen constants at our disposal.

One special way of doing this at once suggests itself. If we reduce the number of constants c_1, c_2, . . . in the specification of Ω to a small finite number, it ought to be possible to satisfy all the equations. I have examined a number of instances of these special simplified universes. The one which is now worked out in detail is much more important than any other I have found, and indeed includes most of the others as special cases.

Consider an arrangement of stars such that

$$\Omega = F(c_0 + c_1x + c_2y + c_3z + c_{11}x^2 + c_{12}xy + \ldots + c_{33}z^2) \quad (26)$$

where F is a function unspecified. This will represent an arrangement of matter for which the equipotential surfaces are the ellipsoids

$$c_0 + c_1x + c_2y + c_3z + c_{11}x^2 + \ldots + c_{33}z^2 = \text{cons.}$$

By a new choice of axes in space the value of Ω may, without any loss of generality beyond that involved in fixing the axes, be put in the form

$$\Omega = \mathrm{F}(\alpha x^2 + \beta y^2 + \gamma z^2 - 1) \quad . \quad . \quad . \quad (27)$$

and on substituting this value for Ω equation (22) becomes

$$4[(b-c)\beta\gamma yz + h\alpha\gamma xz - g\alpha\beta xy + f(\gamma^2z^2 - \beta^2y^2)]\mathrm{F}'' + 2f(\gamma-\beta)\mathrm{F}' + 6\left[\frac{\partial f}{\partial z}\gamma z - \frac{\partial f}{\partial y}\beta y\right]\mathrm{F}' = 0.$$

Short of F having a still more highly specialised value, this equation can only be satisfied by satisfying independently the two equations

$$(b-c)\beta\gamma yz + h\alpha\gamma xz - g\alpha\beta xy + f(\gamma^2z^2 - \beta^2y^2) = 0 \quad . \quad (28)$$

$$f(\gamma-\beta) + 3\frac{\partial f}{\partial z}\gamma z - 3\frac{\partial f}{\partial y}\beta y = 0 \quad . \quad . \quad (29)$$

The second of these equations reduces to

$$(\gamma-\beta)(f_0 + y\mathrm{X}_1 + z\mathrm{X}_2 + 4yz\mathrm{X}_3) + 3\gamma z\mathrm{X}_2 - 3\beta y\mathrm{X}_1 = 0 \quad (30)$$

and this must be satisfied for all values of x, y, and z. On equating the separate coefficients to zero, it is found that the equation can only be satisfied if $\mathrm{X}_1 = \mathrm{X}_2 = 0$, and if either $\beta = \gamma$ or $f_0 = \mathrm{X}_3 = 0$.

There are three equations similar to (30), and for them all to be satisfied we must have

$$\mathrm{X}_1 = \mathrm{Y}_1 = \mathrm{Z}_1 = \mathrm{X}_2 = \mathrm{Y}_2 = \mathrm{Z}_2 = 0 \quad . \quad . \quad (31)$$

together with either

$$\alpha = \beta = \gamma \quad . \quad . \quad . \quad . \quad (32)$$

or, alternatively,

$$\beta = \gamma\,; \quad g_0 = h_0 = 0\,; \quad \mathrm{Y}_3 = \mathrm{Z}_3 = 0 \quad . \quad . \quad (33)$$

or, of course, either of the other alternatives derived from (33) by a cyclic interchange of letters.

After the simplification made possible by equations (31), it is found that equation (28) reduces to

$$(b_0 - c_0)\beta\gamma yz + h_0\alpha\gamma xz + g_0\alpha\beta xy + f_0(\gamma^2z^2 - \beta^2y^2) + (x^2\mathrm{Y}_3 + y^2\mathrm{X}_3 - z^2\mathrm{X}_3 - x^2\mathrm{Z}_3)\beta\gamma yz + \mathrm{Z}_3\alpha\gamma x^2yz - \mathrm{Y}_3\alpha\beta x^2yz + \mathrm{X}_3yz(\gamma^2z^2 - \beta^2y^2) = 0 \quad (34)$$

This again must be satisfied for all values of x, y, z, so that the different coefficients must vanish separately. This requires

$$b_0 = c_0\,; \quad f_0 = g_0 = h_0 = 0 \quad . \quad . \quad . \quad (35)$$

together with

$$X_3(\beta-\gamma)=0\,;\quad Y_3(\gamma-\alpha)\beta+Z_3(\alpha-\beta)\gamma=0 \quad . \quad (36)$$

There are three sets of equations of this type. To satisfy the system of three similar to (35) we must have

$$a_0=b_0=c_0\,;\quad f_0=g_0=h_0 \quad . \quad . \quad . \quad (37)$$

while it is readily found that the set of three similar to (36) are satisfied automatically by either of the alternatives (32) or (33).

From equations (31) and (37) it now appears that ϕ_2, as given by equation (17), must reduce to the form

$$\phi_2=q(u^2+v^2+w^2)-\Sigma X_3(zv-yw)^2 \quad . \quad . \quad (38)$$

In addition to this, for star-streaming to be consistent with the steady state there are two alternatives, one of which must be satisfied. These are as follows:—

I. We may have $\alpha=\beta=\gamma$, in which case the equipotentials $\Omega=$ cons. reduce to concentric spheres, and we are dealing with a universe of perfect spherical symmetry.

II. We may have $\beta=\gamma$ only, in which case the equipotentials $\Omega=$ cons. are similar concentric spheroids, and we are dealing with a universe arranged in spheroidal shells. In this case we must have $Y_3=Z_3=0$ in the law of distribution (38).

In the cases represented by both of these alternatives it is readily found that equations (18)–(20) reduce to

$$q\frac{\partial\Omega}{\partial x}=\frac{1}{2}\frac{\partial}{\partial x}(\log A)$$

and two similar equations, and the law of distribution becomes

$$f=Ce^{-q(u^2+v^2+w^2-2\Omega)+\Sigma X_3(zv-yw)^2} \quad . \quad . \quad . \quad (39)$$

in place of the previous law (25). A special case of this law has been discovered and discussed by Eddington.*

I have worked out various other instances of special laws by this method. Except for very artificially constructed universes, the general result always emerges that for Schwarzschild's law to be consistent with the steady state the universe must be either spherical or spheroidal, excepting in the special case in which the Schwarzian ellipsoid reduces to a sphere, so that no star-streaming occurs at any point of the universe.

It may probably be taken for granted that our universe is neither spherical nor of the special spheroidal type we have had under consideration. It may be taken for certain that star-streaming occurs in our own universe, at least at our particular moment of time and in our particular region of space. Assuming that this star-streaming can be adequately represented by Schwarzschild's law, we are led to the conclusion that our universe cannot be in a steady state. The

* *M.N.*, vol. lxxv., 1915, p. 366.

only alternative would be to suppose that the star-streaming we observe is an exceptional phenomenon peculiar to our own very limited glimpse of the universe, and that, except for this transitory and local irregularity, the rest of the universe is devoid of star-streaming. But when this assumption is made we shall find later that the universe cannot be in a steady state unless it is also spherical, and our universe is certainly not spherical.

Thus everything indicates that our universe cannot be thought of as being in a steady state.

A conclusion as far-reaching as this will hardly be accepted without criticism and further confirmation. The most obvious criticism to be brought against it is that other laws of star-streaming may perhaps represent the observed facts as well as Schwarzschild's law, and that with one of these other laws the difficulty might possibly be resolved. I have accordingly attempted to examine the question by a different and more powerful method of analysis, in which we are by no means limited to Schwarzschild's law.

II. *On the most General Law of Distribution possible for a Cluster or Universe in a Steady State.*

The characteristic equation which is satisfied by the law of distribution f can be expressed in a different form from that in which we previously obtained it.

Let N denote the number of stars enumerated in formula (1), so that

$$N = f du dv dw dx dy dz . \quad . \quad . \quad . \quad (40)$$

Let D/Dt be used as usual to denote differentiation with respect to the time when we follow a star or a group of similarly moving stars in its motion. Then the equation of continuity can be expressed in the form

$$\frac{DN}{Dt} = 0 . \quad . \quad . \quad . \quad . \quad (41)$$

But, by the well-known theorem of Liouville,*

$$\frac{D}{Dt}(du dv dw dx dy dz) = 0,$$

so that equation (41) may be put in the form

$$\frac{Df}{Dt} = 0 . \quad . \quad . \quad . \quad . \quad (42)$$

Since f is a function of u, v, w, x, y, z, and t, the appropriate transformation for D/Dt when operating on f is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} + \frac{du}{dt}\frac{\partial}{\partial u} + \frac{dv}{dt}\frac{\partial}{\partial v} + \frac{dw}{dt}\frac{\partial}{\partial w},$$

* Jeans, *Dynamical Theory of Gases* (2nd edition), p. 73.

and on using this our previous equation (3) is seen to follow at once.

Using the form (42) for the characteristic equation, it is apparent that f must remain constant throughout the motion of any single star. The condition for steady motion is $df/dt = 0$, so that f must be independent of the time and be a function of u, v, w, x, y, z only. Hence we see that for a universe in steady motion the law of distribution f must be such that

$$f(u, v, w, x, y, z) = \text{constant} \quad . \quad . \quad . \quad (43)$$

is a first integral, independent of the time, of the three equations of motion of a star

$$\frac{du}{dt} = X, \quad \frac{dv}{dt} = Y, \quad \frac{dw}{dt} = Z \quad . \quad . \quad . \quad (44)$$

Whatever the nature of the motion, or whatever the arrangement of the universe, one first integral of these equations is always known, namely, the energy-integral

$$E \equiv \tfrac{1}{2}(u^2 + v^2 + w^2) - \Omega = \text{cons.} \quad . \quad . \quad (45)$$

E being the energy per unit mass.

If, as will usually be the case, this is the only first integral of the equations of motion, f must be a function of E, and the law of distribution must be of the form

$$f = \phi(E) = \phi[\tfrac{1}{2}(u^2 + v^2 + w^2 - 2\Omega)] \quad . \quad . \quad (46)$$

In the case now under consideration, in which no first integral exists other than the energy-equation, this value of f will give the most general law of distribution possible. Formula (25), previously obtained, is a special case. It will be noticed that Schwarzschild's ellipsoidal law cannot be included in formula (46), nor indeed can any law which is consistent with the existence of star-streaming; the law of distribution (46) requires that at every point the proper motions of the stars shall favour all directions in space equally.

We must next consider cases in which there is more than one first integral of the equations of motion (44). These will, of course, be analogous to those cases previously discussed in which the universe was specially arranged so that the potential Ω had certain properties.

We need not stop to consider the possibility of the occurrence of the integrals $u = \text{cons.}$, $v = \text{cons.}$, or $w = \text{cons.}$, for a moment's reflection will show that these can have no conceivable application to our universe.

If we put

$$\varpi_1 = zv - yw, \quad \varpi_2 = xw - zu, \quad \varpi_3 = yu - xv,$$

the equations of motion (44) give

$$\frac{d\varpi_1}{dt} = zY - yZ, \text{ etc.}$$

Thus $\varpi_1 = \text{cons.}$ will be an integral if at every point of the universe we have $zY = yZ$; *i.e.* if the resultant force at every point passes through the axis of x. This requires that the universe shall be arranged so that the equipotentials shall all be surfaces of revolution. Thus the universe in general must be symmetrical about an axis. For such a universe the most general law of distribution consistent with the existence of a steady state will be

$$f = \phi(\mathrm{E}, \varpi_1) = \phi[\tfrac{1}{2}(u^2 + v^2 + w^2 - 2\Omega),\ zv - yw] \quad . \quad (47)$$

of which the law of distribution previously found for a spheroidal universe is a special case.

It is possible to have the three integrals of motion

$$\varpi_1 = \text{cons.}, \quad \varpi_2 = \text{cons.}, \quad \varpi_3 = \text{cons.},$$

if the universe is such that at every point

$$\frac{\mathrm{X}}{x} = \frac{\mathrm{Y}}{y} = \frac{\mathrm{Z}}{z},$$

i.e. if the resultant force on every star passes through the centre of gravity of the whole universe. This requires that the universe shall be "centrobaric," so that it must be either a spherical universe or a universe in which all the stars remain for ever in one plane. In the former case the most general law of distribution is

$$f = \phi(\mathrm{E}, \varpi_1, \varpi_2, \varpi_3) \quad . \quad . \quad . \quad (48)$$

of which the law (38), previously found, is a special case. In the latter case, if the stars are all in the plane $x = 0$, the most general law of distribution possible is obtained by putting $x = 0$ and $u = 0$ in formula (48), and the law of distribution then reduces to that given by (47).

With one exception I have not succeeded in discovering any universe, no matter how artificial, in which other possibilities occur beyond those already discussed. The one exception is a universe in which the components of force are of the forms

$$\mathrm{X} = \mathrm{R}_1 x, \quad \mathrm{Y} = \mathrm{R}_2 y, \quad \mathrm{Z} = \mathrm{R}_3 z.$$

In this universe there are three integrals of motion of the type

$$u^2 + \mathrm{R}_1 x^2 = \text{cons.},$$

and the most general law of distribution is of the form

$$f = \phi(\mathrm{E}, u^2 + \mathrm{R}_1 x^2, \quad v^2 + \mathrm{R}_2 y^2, \quad w^2 + \mathrm{R}_3 z^2).$$

It may be noticed that for this universe Ω is given by

$$\Omega = \tfrac{1}{2}(\mathrm{R}_1 x^2 + \mathrm{R}_2 y^2 + \mathrm{R}_3 z^2);$$

it is an ellipsoidal universe of uniform density throughout. Moreover, it is included in the special universes already considered, in which Ω has very restricted values.

This universe, and others of similar properties, if they exist, are of such a very artificial type that they may safely be neglected in a discussion of actual possibilities.

Our universe is certainly not spherical, so that the possibility of a law of distribution such as (41) may be left out of consideration. On the other hand, our universe appears to approach sufficiently near to a state of symmetry about an axis for the law of distribution (47) to be deserving of a more detailed examination.

Let us take the ordinary cylindrical co-ordinates x, ϖ, θ, the axis of x being as before the axis of symmetry and ϖ^2 being equal to our former y^2+z^2. Let U, V, W be used to denote the components of velocity in the directions respectively of x, ϖ, and θ increasing. Then we have

$$U=u, \quad V=\frac{yv+zw}{(y^2+z^2)^{\frac{1}{2}}}, \quad W=\frac{zv-yw}{(y^2+z^2)^{\frac{1}{2}}}$$

and the law of distribution (47) becomes

$$f=\phi[\tfrac{1}{2}(U^2+V^2+W^2-2\Omega),\ W\varpi] \quad . \quad . \quad (49)$$

At any particular point in space this law of distribution of velocities alone is of the form

$$f=\phi(U^2+V^2,\ W) . \quad . \quad . \quad . \quad (50)$$

showing that the law of distribution is an even function of U and V and involves U and W only through U^2+V^2. It will be noticed that $(U^2+V^2)^{\frac{1}{2}}$ is the velocity in any meridional plane, so that the components of velocity in each meridional plane are distributed evenly as regards directions in this plane. A law of distribution of the form of (50) can only be reconciled with an ellipsoidal law of distribution in the special case in which the ellipsoid reduces to a spheroid of which the axis of revolution points in the direction of θ increasing; a condition which does not appear to be satisfied in our own universe.

III. *On the Dynamical Equilibrium of a Universe of Stars.*

With any law of distribution f, the density of stars at any point is given by

$$\nu=\iiint f\,du\,dv\,dw \quad . \quad . \quad . \quad . \quad (51)$$

If the law of distribution satisfies the condition $df/dt=0$, then $d\nu/dt=0$ at every point, so that the universe will retain its shape and distribution of density through all time. Thus all permanent arrangements of a universe must be derivable from solutions of equation (51) which make f independent of the time, ν and Ω being connected by the usual relation $\nabla^2\Omega=-4\pi\nu m$. Eliminating ν, the equation becomes

$$\nabla^2\Omega=-4\pi m\iiint f\,du\,dv\,dw \quad . \quad . \quad . \quad (52)$$

and values of f which satisfy this equation and make the right-hand member of equation (3) vanish will give permanent universes.

The equations of motion of any small portion of a universe are readily found to be *

$$\frac{\partial}{\partial t}(\nu\bar{u}) + \frac{\partial}{\partial x}(\nu\overline{u^2}) + \frac{\partial}{\partial y}(\nu\overline{uv}) + \frac{\partial}{\partial z}(\nu\overline{uw}) = \nu\frac{\partial\Omega}{\partial x} \quad . \quad (53)$$

and two similar equations. Here a bar over any function of u, v, w denotes the mean value of the function for all stars near the point x, y, z. These equations are directly deducible from the equations of motion (44), or they can also be deduced from equation (51) in conjunction with equation (3). Thus equation (51) may be regarded as an integrated form of the equations of equilibrium (53).

To search for permanent universes, we must examine under what conditions the solutions already obtained for f can also be made to satisfy equation (52). We may consider in turn the three separate forms for f given by equations (46), (47), and (48).

If f is of the form given by equation (46), namely,

$$f = \phi(\mathrm{E}) = \phi[\tfrac{1}{2}(u^2 + v^2 + w^2) - \Omega],$$

it is clear from equation (52) that $\nabla^2\Omega$ must be a function of Ω only. This can only be satisfied by the equipotentials being concentric spheres, so that the universe can only remain permanently in equilibrium if it is spherical.

When the law of distribution is of the type given by equation (48), the universe must again be spherical, as we have already seen.

Thus the only possible law of distribution that can give a permanent non-spherical universe is that given by equation (47), namely

$$f = \phi(\mathrm{E}_1\varpi_1) = \phi[\tfrac{1}{2}(u^2 + v^2 + w^2) - \Omega,\ zv - yw] \quad . \quad (54)$$

A special case included under this law is that of a universe rotating as a whole and so having the shape of a figure of revolution. Other special cases can be obtained by superposing two rotating universes having the same axis of rotation but rotating in opposite directions. In such a case as this star-streaming obviously occurs, but in every case covered by equation (54) the star-streaming must occur in circles round the axis of symmetry of the universe.

In no way, then, does it appear possible to regard our universe as one in which steady motion is permanently established.

IV. *Summary and Conclusion.*

In the first part of this paper we examined under what conditions Schwarzschild's ellipsoidal law was possible in a universe in a steady state. A true ellipsoidal law was found to be possible

* *Cf.* Jeans, *Dynamical Theory of Gases*, 1st edition, equation (358), or 2nd edition, equation (454).

only in a universe which is of spherical shape. When the universe is not spherical, but possesses symmetry about an axis, the Schwarzschild ellipsoid degenerates into a spheroid, the two axes in the meridional plane of the universe becoming equal. In all other cases, except possibly a few special and highly artificial ones, the Schwarzschild ellipsoid reduces to a sphere, so that the proper motions of the stars occur in all directions equally, and there can be no star-streaming.

In the second part of the paper the restriction to laws of ellipsoidal form was abandoned, and an attempt was made to discover the most general law of distribution which can be consistent with a steady state. In general, when a star moves under the gravitational influence of the whole remainder of the universe, there will be only one first integral of its equations of motion independent of the time, namely, the energy integral. When this is the case, it is found that the velocity diagram for every region in space must reduce to a sphere, so that there can be no star-streaming anywhere. Thus, if star-streaming is observed in a universe which is supposed to be in a steady state, it can only be because every star has, in addition to the energy integral, another first integral which is independent of the time.

Except for highly artificial arrangements, such as are hardly likely to have any relations to actuality, additional first integrals can only be obtained by supposing the universe to have at least one axis of symmetry of figure. Our universe may with some confidence be supposed not to be spherical, so that the only case of importance to us is that of a universe shaped like a figure of revolution. In such a universe there is always one additional first integral, namely, an integral expressing constancy of the moment of momentum about the axis of the universe. In such a universe we found the most general law of distribution of velocities which could be consistent with the existence of a steady state. This law was found to be consistent with the existence of star-streaming, although not of star-streaming of the kind which is observed in our own universe. Thus even if our universe may legitimately be thought of as possessing symmetry about an axis of revolution, it appears that star-streaming such as is observed in our universe does not seem to be consistent with the supposition that our universe is in a steady state.

In the third section of the paper, we examined the condition that the different parts of a universe should be dynamically in equilibrium. It was found that a universe in which the velocity diagram is everywhere spherical can only be in equilibrium when its own shape is spherical. It was found that the only universe which can be in equilibrium in a shape other than spherical is one in which star-streaming occurs in circles around the axis of the universe, the universe itself being symmetrical about this axis. It is doubtful whether our universe may legitimately be supposed to have this kind of symmetry, but in any case the observed star-streaming is not of the kind required, so that we have to

suppose that our universe cannot be dynamically in a state of equilibrium.

We are therefore compelled to give up all hope of unravelling the mechanism of our universe by assuming that it is in a steady state, and using the phenomenon of star-streaming as a means of exploring its structure. It ought perhaps to be noticed that there was never any valid reason for supposing that we should arrive at the truth by making the assumption that the universe is in a steady state. Indeed, the *a priori* probabilities are all in the other direction. The direct observational knowledge which we have of the movements in our universe is excessively limited both in time and space: the introduction of the steady-state hypothesis is only a tentative effort to remove the restriction in time. In the same way a being whose life was infinitely more restricted than ours might try if he could explain such phenomena as an earthquake or a lightning flash in terms of a steady-state hypothesis. His attempts, as we know, would be foredoomed to failure, and the results we have obtained seem to indicate that attempts to explain star-streaming as a steady-state phenomenon must be equally foredoomed to failure. It appears that we must not read into star-streaming anything more than we can actually see, or perhaps at most we may regard it as a motion of parts of the universe which are in process of taking up the positions or motions they will ultimately have in some future steady state which has not yet been attained.

The Viscosity of the Earth. (*Second Paper.*)

By Harold Jeffreys, B.A., M.Sc.

As Sir Joseph Larmor has been kind enough to send me the gist of his note before publication, I take this opportunity of partially answering it and adding some further notes of my own.

The law of elastico-viscosity used by Sir G. H. Darwin and myself* was originally given by Mr. J. G. Butcher.† With the physical assumptions he makes, the law is exact whatever the value of pt, and my treatment is sound. To what extent these hold is, of course, quite another matter. The discrepancy will, however, have to be very serious to remove the difficulty I raised, as the observed life of the variation of latitude is at least a thousand times what I derived from the theory of tidal lag. In a paper at present in the possession of the London Mathematical Society I have given reasons for believing that non-linearity in the law of viscosity is not likely to help to reconcile the two theories. I think that any law capable of doing so would have to be so complicated qualitatively as to be even more improbable than the Butcher law.

* *Monthly Notices*, vol. lxxv., 1915, p. 648.
† *Proc. Lond. Math. Soc.*, 1876 Dec. 14, pp. 107–9.

To put the matter in another way, suppose that a body is strained by external forces of such magnitude as to keep at every point constantly the same strain as the time varies. Then if the elastico-viscous law holds, after an interval t the stresses required will have diminished to e^{-1} of their original value, the solid having flowed in the meantime. After a time $2\pi t$ they will have decreased to $\frac{1}{500}$ of their initial value, so that the body will be nearly in a hydrostatic state. In the case of the earth, using the value of t given by the lunar secular acceleration, we find $2\pi t$ = 10 days roughly. Now it seems to me that such a gradual approximation to the hydrostatic state will be closely imitated in any solid undergoing permanent set; such set must occur if bodily tidal friction is to exist. To take an estimate that seems distinctly large, suppose that set takes place in such a way that the hydrostatic form is nearly attained in 60 days. Then in one period of a vibration lasting 430 days, there is time for the hydrostatic form to be taken seven times, so that the body will behave nearly like a perfect liquid; in this such a vibration as a Eulerian nutation is known to be impossible.

In order that a Eulerian nutation may be possible, then, it seems necessary that the time required for appreciable permanent set to take place must be several years; and in such an earth bodily tidal friction must be very small in amount, unless the law governing the set is something very remarkable.

At this point I may mention that in Sir G. H. Darwin's *Collected Papers*, vol. ii. p. 17, where this law is quoted, in the value of P he gives, $\delta/3t$ is misprinted $\alpha/3t$.

For the above reasons, then, I think the theories of tidal lag as an explanation of the secular acceleration, and of the Eulerian nutation as an explanation of the 430-day period, are mutually inconsistent, although I admit there is some room for personal opinion in the argument, and am myself open to conviction. Neither theory is wholly satisfactory by itself, that of tidal lag being perhaps slightly the worse of the two, since the observed secular acceleration of the moon is only $1\frac{1}{2}$ times that of the sun, whereas the theoretical ratio is 5. My impression is that these accelerations should be regarded as empirical terms without known *vera causæ*, and probably connected with the irregular fluctuations recently discussed by Sir Joseph Larmor and Mr. H. Glauert.

Oceanic viscosity does not help, as I showed in the June *Monthly Notices* that the tidal lag in an ocean of four miles' depth is more than 10,000 times too small to account for the lunar acceleration. The fact that the ocean actually covers only part of the earth scarcely seems likely to account for such a factor as this.

I stated in June that nothing in my paper had any reference to the theory of tidal friction as applied to the past history of the earth and moon. I have since seen that it may actually be used to strengthen this theory. Sir G. H. Darwin based his estimate of the minimum time taken for the changes on a hypothetical variable viscosity that caused the tides to lag by a constant angle

$22\frac{1}{2}^\circ$, thus making the speed of the changes a maximum. This optimum viscosity would be rather more than 10^{14} C.G.S. units soon after the commencement of the evolution, and would steadily increase till at present it would be nearly 10^{15}. It is easily seen to be proportional to the length of the lunar day. Now, when the earth was liquid the viscosity must have been small. At present, however, the earth seems to be nearly perfectly elastic, the viscosity being about 10^{17} or ∞ according as we accept or reject the use of the tides as an explanation of the secular acceleration. In either case, therefore, the actual value must have been equal to the optimum value at some intermediate epoch. Further, as both must have increased with the time, they may have been nearly equal for some considerable period. We are thus left with the suggestion that the earlier part of the evolution took place very slowly, and that the earth possibly solidified when the moon's distance had increased by comparatively little. The evolution would next become very rapid over an interval of the order of 10^8 years, and then the system would remain in a practically invariable state over a great and indeterminate period reaching to the present day. As the optimum value is so great, it corresponds to a solid body, so that the most rapid part of the evolution must certainly have occurred since solidification.

The Figure of the Earth : a Reply to Mr. Hinks.

By Harold Jeffreys.

If $f(\theta, \phi)$ be a function of position on a sphere, then in general it is possible to express it in the form of the sum of an infinite series of spherical surface harmonics of ascending orders, so that

$$f(\theta,\phi) = a_0 + a_1 S_1 + a_2 S_2 + a_3 S_3 + \ldots + a_n S_n + \ldots$$

Although in some respects this expansion resembles a least-square approximation, yet in one most important particular it differs essentially from it. It can be applied equally well whether the values of $f(\theta, \phi)$ are distributed according to the error law or not, for the determination of the coefficients depends only on the computation of integrals of the type $\iint f(\theta, \phi) S_n dS$ over the sphere, and then the series found is *exactly* equal to the function $f(\theta, \phi)$. There is no question of residuals or of probable error in the solution.

In the part of my memoir that deals with the earth, I was considering solely that particular surface harmonic known as P_2, equal to $\frac{3}{2}\cos^2\theta - \frac{1}{2}$. On account of the fact that all the equations involved are linear, it is possible to treat every harmonic separately, none of the others interfering with the treatment of a particular term. Consequently, when I had shown that to a sufficient degree of accuracy the coefficient of P_2 in the elevation of the lithosphere

over a sphere of equal volume was equal to the coefficient of P_2 in the elevation of the hydrosphere over a sphere of equal volume, I could safely substitute this value in the Herglotz equations, and all the subsequent mathematical work is quite unaffected by the other harmonics that may occur in the equation of the lithosphere.

Passing to the question of what physical meaning can be attached to the coefficient of P_2 thus found, I wish to point out that this harmonic is fundamentally different physically from all the others. It is the only one with a permanent known *vera causa*. The others are essentially transient in character. Nearly all the great mountain chains are very largely of Tertiary age; such are the Alps, the Carpathians, the Apennines, the Balkans, the Rocky Mountains, and the Himalayas.* Apparently the oldest of them is the Altai, which is not earlier than the Trias;† the Andes† are mostly volcanic, with exposed sedimentary rocks of various ages from Cambrian to Tertiary. Some rather smaller ranges, such as the Urals and the British mountains, are Palæozoic, but the amount of contortion in some of them certainly suggests that even they may have been below the sea since they were formed, and that their apparent age is due to the removal of more recent deposits by denudation. The greater part of the continental masses must have been submerged several times in geological history, owing to fluctuating subterranean causes. My object is to disentangle from the effects of these the much more slowly varying part due to the earth's rotation. This is constant in sign, and steadily diminishing on the theory of tidal friction; it is proportional to $P_2(\cos\theta)$. The way to determine this completely would be to find this part by spherical harmonic analysis and then to remove the more rapidly variable part of it by integrating with regard to the time over several geological periods. From the nature of the case this is, however, impossible. We must therefore work with the present coefficient alone, and assume that the impermanent part of it is small; this is quite likely on account of the wide area affected by this harmonic, as geological upheaval seems to be to some extent a local phenomenon.

I think the above contains the answer to the greater part of Mr. Hinks's Questions 4 and 5. My whole point is that all the permanent part of the deviation of the lithosphere from the geoid is included in P_2, and that the inequalities suggesting that the figure of the earth is different from a spheroid are eliminated in nature by averaging over a great time as effectively as they are in my treatment by the process of harmonic analysis.

Question 1 refers to my alleged neglect of isostasy in allowing for the weight of the ocean. The question whether this theory can be applied to any part of the permanent P_2 harmonic cannot be answered from the present observational evidence, but I think such an application would not be justified. The stress differences due to this harmonic would reach a maximum at the centre of the

* Geikie, *Text-book of Geology.*

† Suess, *The Face of the Earth.*

earth, so if any adjustment is going to occur it will probably start there. The maximum stress difference in the case of a series of mountain ranges is at a comparatively small depth, and as the cause of their upheaval also is probably fairly near the surface, there is some *a priori* evidence that isostasy may exist for them. The state of affairs for P_2 is totally different. For this reason I did not include isosatic compensation in the crust among the possibilities to be considered. It does not, however, make much difference in any case. I write from memory, being at present two hundred miles from the calculation, but when I did the same work without introducing the factor $\frac{7}{11}$, I believe I got an ellipticity with a negative sign and five figures in the denominator, as in the case I published.

Question 2 concerns my definition of ellipticity. In geodesy the term refers to the spheroid of equal volume that most closely resembles the hydrosphere. The elevation of this above the sphere of equal volume can be put in the form $aP_2(\cos\theta)$. If the elevation of the hydrosphere is $f(\theta, \phi)$, the geodetical method is to find a, so that $\int\int(f - aP_2)^2 dS$ over the sphere is a minimum. This leads at once to $a\int\int P_2^2 dS = \int\int f P_2 dS$.

The value of a thus obtained is exactly that obtained by harmonic analysis, and my definition of the ellipticity as $-\frac{3}{2}a$ is therefore identical with the geodetic definition. Similar arguments hold for the lithosphere. The fact that the geoid is not an accurate sphere does not matter in finding the difference of ellipticity between it and the lithosphere from the differences in height, as the error is only of the order of the square of the deviations, or 10^{-5}.

In answer to Question 3, the heights in the table were taken from Messrs. W. & A. K. Johnston's large-scale contoured maps of the continents and oceans, dated 1909, so that they are comparatively modern. The values for the poles were not included in the table, partly because, with due deference to Mr. Hinks's opinion, they are not so well known as the others (he gets them by extrapolation), and partly because they did not matter anyhow, since it was obvious that they would be multiplied by zero in the course of the numerical integration. The reason for the queried figures along the parallel of 80° S. is that this runs so nearly along the edge of the antarctic continent, fringed with floating ice, that in many cases it is difficult to be sure whether a particular point represents land or sea.

Mr. Hinks's criticism of the mode of distribution of the points in my table could be applied with equal validity to any other investigation where the same method has been employed; and they are many. I do not think he can suggest a better one, and if he tries to have "any regard at all to the actual configuration of the land or the ocean," he will run serious risk of introducing a large systematic error. The way to get greater accuracy would be to use a network of the same type with smaller meshes, but for the purpose in question this is unnecessary. Thus, to show what the

effect of the neglect of the sadly ill-treated Cordillera is likely to be, suppose the whole area of one of the meshes at the equator to be raised by 18,000 feet. The coefficient of P_2 would be diminished by less than 200 feet, and 1000 feet is almost certainly a greater change than all of the errors (even if cumulative, which they are not) involved in the method of sampling could possibly produce.

In the last paragraph under Question 3 Mr. Hinks gives no reference to Murray to show where he obtains his percentages of 35 and 65 for the areas of the ocean respectively shallower than and deeper than 12,000 feet, and the figures he quotes from "The Depths of the Ocean" later in the paper correspond to 41 per cent. and 59 per cent. respectively. The rest of the discrepancy is probably due to his method of combining my heights. He simply counts up the points where the depths are greater and less than 12,000 feet, and treats all as of equal weight in finding the areas, completely leaving out of account the fact that the compartments represented by these points are not of equal area. The consequence of this procedure is that the shallower areas in high latitudes are greatly exaggerated in importance.

Mr. Hinks devotes so much attention to quite small possible causes of error that I doubt whether he realises the magnitude of the difference of ellipticity that would be required to vitiate my main conclusion: that the earth must have readjusted itself to the hydrostatic form since it solidified. On pp. 204–5 of my memoir I gave reasons for believing that the earth solidified when the quantity $\frac{1}{2}an^2/g$, to which the hydrostatic ellipticity is proportional, was $\frac{1}{111}$, whereas it is now $\frac{1}{582}$. Even taking its value at $\frac{1}{200}$ and applying a factor $\frac{1}{3}$ to the difference to allow for elastic yielding, and a further $\frac{5}{2}$ to give the ellipticity, we have,

$$\text{theoretical difference in ellipticity} = \tfrac{1}{3} \cdot \tfrac{5}{2} \left(\tfrac{1}{200} - \tfrac{1}{582} \right) = \tfrac{1}{366}.$$

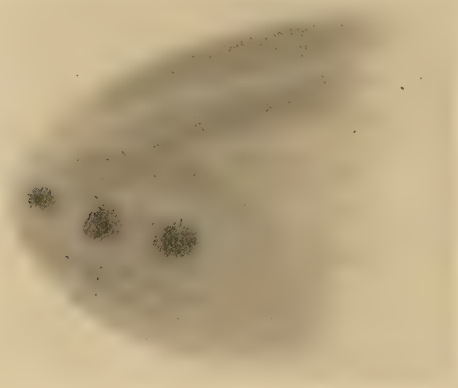
This will have to be somewhat reduced on account of the attraction of this mass on the ocean, which will induce some ellipticity in the hydrosphere. We are, however, probably on the safe side in taking the final value as greater than $\frac{1}{300}$. This corresponds to a broad strip of elevated ground running right round the equator, 5 miles higher than at either of the poles. This bears little resemblance to the actual earth.

I claim to have shown, then, that if my theory were false the difference of ellipticity determined by my method should be $> +\frac{1}{300}$. The observed value may safely be said to lie between $+\frac{1}{30000}$ and $-\frac{1}{10000}$.

Note on Comet Mellish (1915 *a*) 1915 *October* 4.
By F. Henroteau, D. ès Sc.

(*Communicated by the Rev. A. L. Cortie, S.J.*)

Comet Mellish (1915 *a*) was photographed at this observatory with the Whitelow 6-inch camera on 1915 October 4, with an exposure from $16^h\ 5^m$ to $16^h\ 55^m$ G.M.T. The image on the plate



Comet Mellish (1915 *a*).
Enlarged drawing from the original photograph by F. Henroteau.

is faint, but an enlarged drawing has been faithfully made from it with the aid of a microscope. The appearance of the comet is reproduced in the drawing annexed. A careful examination by Father Cortie and myself shows the existence of three nuclei. The separations of the nuclei from means of a series of measures taken by both of us are as follows: nucleus 1 to nucleus 2, 18″; nucleus 2 to nucleus 3, 25″. The scale of the plate was obtained from the reference stars 17 Leporis and β Canis Majoris.

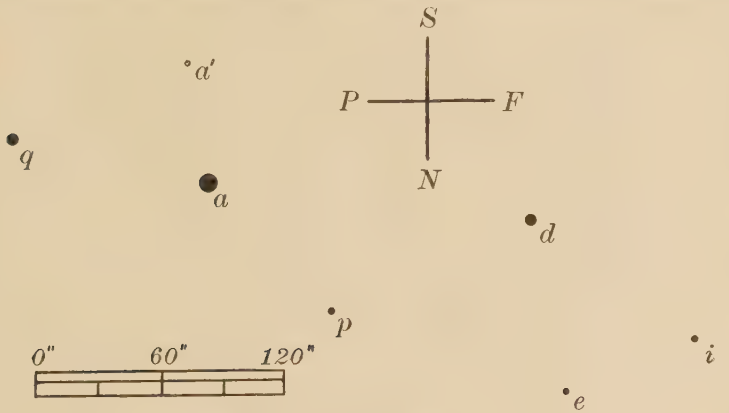
Stonyhurst College Observatory.

Measures of Small Stars in the Dumbbell Nebula (= *N.G.C.* 6853 = *M* 27) 1860·0 α $19^h\ 53^m\ 33^s$, $\delta + 22°\ 20'·4$. By E. E. Barnard.

In 1851 Otto Struve measured a number of small stars in the Dumbbell Nebula with the micrometer of the Pulkova refractor, with a magnifying power of 207 and illuminated wires. These observations are printed in *Philosophical Transactions*, vol. cli., part iii., pp. 738–740. Comparatively few distances were measured —these quantities being computed from the position angles and

other measured distances. Evidently Struve considered the results satisfactory, for he says, "I have got the conviction that the probable error in any of the following co-ordinates [computed $\Delta\alpha$ and $\Delta\delta$] will be considerably inferior to one second of space. It will be comparatively smaller for the brighter stars, but somewhat greater for the very faint objects." He gives a table of computed $\Delta\alpha$ and $\Delta\delta$ for these observations.

In 1894 Rambaut and Wilson measured these same stars on two photographs taken by the latter on 1893 September 3 and 8, with exposures of 27 minutes and 2 hours respectively (*M.N.*, vol. liv. p. 319). They considered their results exact to within two- or three-tenths of a second of arc. A comparison of their measures with those of Struve showed large discordances, amount-



Small Stars in the Dumbbell Nebula.

ing to as much as 5″ or 6″, which they attributed to motion in the small stars.

In *Astronomy and Astrophysics*, vol. xiii., for June 1894, p. 445, I have given measures of a few of these stars made with the 36-inch refractor of the Lick Observatory in 1894, which seemed to confirm the photographs.* In *Monthly Notices*, vol. lxii. p. 466, I have given later measures, which were made with the 40-inch telescope of the Yerkes Observatory. These seemed to negative motion in the few stars that were measured. I have lately repeated these measures, and the results seem to show that no considerable motion exists in the stars under consideration. They agree quite closely with Struve's results except in two cases where the position angles are discordant. One of these, I feel sure, is due to a mistake in the original measures. The other may be due to motion. There is also a possible change in the distance of *d*.

* In *Astronomy and Astrophysics*, vol. xiii., at the top of p. 447, the mean of the computed $\Delta\delta$ for star *d* should be 16″·03 instead of 16″·37. This makes a slight correction necessary in the comparisons for *d* on the same page. In the diagram on p. 446 the north and south should be interchanged.

Following are the present measures. They are double distances, as were my previous measures. I have retained the original notation of Struve. Unless atmospheric conditions are good, these stars are difficult because of their position in the nebulosity.

Measures of the Small Stars in the Dumbbell Nebula.

				Mag.	Mag.
		$a-q$.			
1915·391	May 23	257°·26	97″·34	...	12·9
·418	June 2	257·17	97·46	...	...
·454	15	257·15	97·18	...	...
1915·421		257·19	97·33	...	12·9
		$a-a'$.			
1915·391	May 23	189°·30	58″·53	...	13·5
·418	June 2	189·05	58·66	...	...
·454	15	189·01	58·59	...	...
1915·421		189·12	58·59	...	13·5
		$a-p$.			
1915·391	May 23	43°·45	83″·80	...	13
·418	June 2	43·25	83·93	...	...
·454	15	43·68	83·92	...	...
1915·421		43·46	83·88	...	13
		$a-d$.			
1915·391	May 23	83°·73	154″·60	11·7	13
·418	June 2	83·62	154·59	...	...
·517	July 8	83·57	154·30	...	...
·530	13	83·78	154·43	..	...
1915·464		83·68	154·48	11·7	13
		$a-e$.			
1915·473	June 22	59°·59	199″·43	...	...
·517	July 8	59·55	199·08	...	...
·550	20	59·46	199·01	...	...
1915·513		59·53	199·17	...	...
		$a-i$.			
1915·574	July 29	72°·17	248″·04	...	...
·607	Aug. 10	72·05	248·11	...	...
·626	17	71·95	248·24	...	...
1915·602		72·06	248·13	...	...

Struve's original measures were made in position angle and distance. All my measures were made by the same method. Rambaut and Wilson used rectangular co-ordinates, and their comparisons are given in $\Delta\alpha$ and $\Delta\delta$. I have, therefore, reduced

their measures to position angle and distance for comparison, and have also re-reduced Struve's back to angle and distance where necessary.

Comparison of Observations.

				Comp. with OΣ.		
				Δ*p*.	Δ*s*.	
			a – *q*.			
OΣ	1851	257°·9	99″·8	...	...	
R – W	1893	257·7	96·6	+0·2	+3·2	
B	1894	257·2	97·1	+0·7	+2·7	
B	1898	257·3	97·3	+0·6	+2·5	
B	1915	257·2	97·3	+0·7	+2·5	
			a – *a′*.			
OΣ	1851	192°·2	58″·5	...	...	
R – W	1893	189·5	58·3	+2·7	+0·2	
B	1894	189·5	58·3	+2·7	+0·2	
B	1898	189·4	58·4	+2·8	+0·1	
B	1915	189·1	58·6	+3·1	−0·1	
			a – *p*.			Σ's corr'd P.A.
OΣ	1851	46°·3 *	83″·8	...	...	...
R – W	1893	43·3	82·6	+3·0	+1·2	+0·4
B	1894	43·8	83·6	+2·5	+0·2	−0·1
B	1915	43·5	83·9	+2·8	−0·1	+0·2
			a – *d*.			
OΣ	1851	85°·2	153″·0	...	...	
R – W	1893	83·7	152·4	+1·5	+0·6	
B	1894	84·0	154·1	+1·2	−1·1	
B	1898	83·7	154·3	+1·5	−1·3	
B	1915	83·7	154·5	+1·5	−1·5	
			a – *e*.			
OΣ	1851	60°·3	199″·4	...	...	
R – W	1893	59·4	197·1	+0·9	+2·3	
B	1915	59·5	199·2	+0·8	+0·2	
			a – *i*.			
OΣ	1851	73°·3	248″·7	...	...	
R – W	1893	72·3	244·5	+1·0	+4·2	
B	1898	72·2	248·0	+1·1	+0·7	
B	1915	72·1	248·1	+1·2	+0·6	

* It is more than probable that in Struve's P.A. the complement of the angle has been used by mistake, and that the P.A. should be 43°·7. This would make his observation agree closely with that of 1915.

Struve measured position angle and distance for stars *e* and *q*, but for *a'* and *p* position angles only. He did not refer *i* to *a* at all, but determined the position indirectly through *e* and *h*. No correction for precession has been applied to Struve's angles. The change from this cause at the present time would be $-0^{\circ}\cdot3$.

I have thought it worth while to determine the position of the comparison star *a*. For this purpose the following observations were made on three nights of this year:—

a – B.D. $+22^{\circ}$ 3881 ($9^{m}\cdot0$)	a – B.D. $+22^{\circ}$ 3887 ($7^{m}\cdot6$)
$\Delta\alpha = -0^{m}\,40^{s}\cdot35$ (20 tr., 1 night)	$\Delta\alpha = -1^{m}\,18^{s}\cdot36$ (20 tr., 1 night)
$\Delta\delta = +0'\,41''\cdot2$ (14 meas., 3 nights)	$\Delta\delta = +0'\,6''\cdot1$ (13 meas., 3 nights)

The star $+22^{\circ}$ 3887 has a considerable proper motion in declination. From an investigation of the various catalogues, I have deduced for it an annual proper motion of $-0''\cdot28$ in δ. There does not seem to be any certain motion in α. Applying this value to the declinations, the following places are obtained:—

Authority.	Epoch of Obs.	α 1915·0.	δ 1915·0.
		h m s	° ′ ″
Munich 22661	1844·6	19 57 3·81	+22 28 53·6
Bonn +22° 3887	1862·2	19 57 3·57	+22 28 54·3
Paris 27248	1879·7	19 57 3·51	+22 28 54·9
Berlin A.G.C. 7419	1881·7	19 57 3·53	+22 28 54·1
Porter 1699	1895·6	19 57 3·56	+22 28 54·3
	Mean =	19 57 3·60	+22 28 54·2
	To epoch 1915·62		− 0·2
			+22 28 54·0

The Munich *a* is discordant, but I have not seen fit to discard it.

These give the position of the star *a*:

$$1915\cdot0 \quad \alpha\ 19^{h}\ 55^{m}\ 45^{s}\cdot24 \quad \delta + 22^{\circ}\ 29'\ 0''\cdot1$$

This star (*a*) is in the preceding edge of M 27, at the north end of a thin continuation of the bright outline of the nebula, which abruptly terminates at this point, but continues in a fainter form as shown in the various photographs. According to these photographs the star *d* is perfectly central in the bright part of the nebula.

The position of $+22^{\circ}$ 3881 is discordant when brought up from the several catalogues in which I have found it. It is thought best to determine anew its position from the above observations.

$$(+22^{\circ}\ 3881) - (+22^{\circ}\ 3887)$$

$$\Delta\alpha = -0^{m}\ 38^{s}\cdot01\ (20\ \text{tr.}) \quad \Delta\delta = -0'\ 35''\cdot1\ (8\ \text{meas.})$$

These give for B.D. + 22° 3881 :

1915·0 α 19^h 56^m 25^s·59 δ + 22° 28′ 18″·9

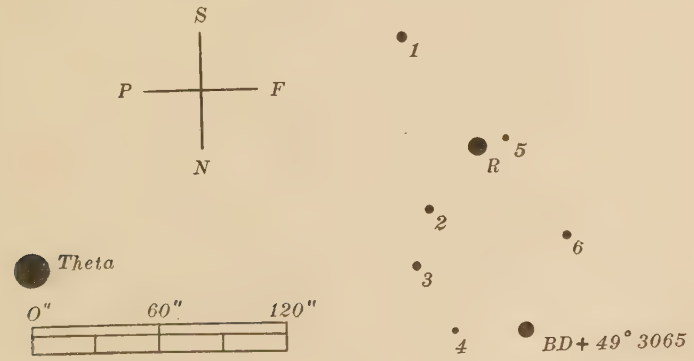
The position of this star by Sievers in *A.N.*, vol. lviii. p. 372, = Kam 2924*w*, is certainly wrong by over 2^s in α and some 10″ or more in δ. There is perhaps some motion in both co-ordinates.

The photograph of this nebula in the *Publications of the Lick Observatory*, vol. viii. (plate 60), is oriented with *north* up, and not *south* as stated on p. 46 of the index to that volume.

Yerkes Observatory, Williams Bay, Wisconsin:
1915 *August* 20.

Measures of the Positions of Faint Comparison Stars near the Variable R Cygni. By E. E. Barnard.

In vol. lxxiii. p. 30 of the *Monthly Notices of the Royal Astronomical Society* I have given some observations of R Cygni when it was faint. The diagram accompanying that paper showed some faint stars near the variable whose places had only been estimated. The star is now bright, and I have lately measured these small stars (including the star B.D. + 49°3065) with the



Small Stars near R Cygni.

micrometer of the 40-inch refractor, as they may be valuable in the future, especially for observations of R Cygni in its fainter stages. They are referred to the variable as origin.

B.D. + 49°3065 and R Cygni.

1915·391	May 23	193°·80	91″·08
·418	June 2	193·79	91·10
1915·404		193·80	91·09

The previous measures gave:

1912·741 193°·81 91″·09 (3 nights).

Star 1.

1915·418	June 2	212°·76	65″·30
·443	11	212·71	65·53
·454	15	212·77	65·21
1915·438		212·75	65·35

Star 2.

1915·418	June 2	321°·78	37″·35
·443	11	321·66	37·32
·454	15	322·23	37·45
1915·438		321·89	37·37

Star 3.

1915·418	June 2	333°·14	65″·56
·443	11	333·00	65·51
·454	15	333·33	65·61
1915·438		333·16	65·56

Star 4.

1915·418	June 2	352°·68	88″·18
·443	11	352·73	88·38
·454	15	352·70	88·48
1915·438		352·70	88·35

Star 6.

1915·418	June 2	43°·42	60″·85
·443	11	43·57	61·12
·454	15	43·70	61·24
1915·438		43·56	61·07

Star 5 was not measured because of the brightness of the variable. The position, however, was previously measured.

Star 5.

1912·790 Oct. 15 108°·4 14″·63 (1 night),

and its magnitude given as 15. The measure, however, is not considered good.

An estimate of the brightness of these objects was made on June 2.

1 = 12·2 magnitude
2 = 13·9 ,,
3 = 14 ,,
4 = 14·2 ,,
6 = 14·5 ,,

Following are the previous estimates of their brightness.

1 = 12 magnitude
2 = 13·3 ,,
3 = 13·7 ,,
4 = 13·7 ,,
5 = 15 ,,
6 = 13 ,,

The greatest discordance in the two sets of estimates is in the star 6, where the difference is over one magnitude.

A few observations have been obtained of the variable in its present brighter condition. These were made with the 4-inch finder of the 40-inch telescope.

R Cygni in 1915.

	d	h				d	h		
May	23	18	G.M.T.	7·5 mag.	June	22	17	G.M.T.	8·2 mag.
June	2	17	,,	7·7 ,,	July	13	16	,,	8·0 ,,
,,	11	17	,,	7·9 ,,	,,	20	17	,,	8·5 ,,
,,	15	17	,,	8·4 ,,					

These magnitudes are on the B.D. scale. I think the discordance in the results is due to errors in the observations.

The Colour of the Variable.

The star was a deep red at times, but the depth and purity of the colour of a star in the large telescope depends much on the conditions of seeing. The out-of-focus image had a red core, but there was no definite focus for this red colour. The regular focus did not differ from that of an ordinary star.

The position of R Cygni given in *M.N.* lxxiii. from the previous measures was:

1912·0 $\alpha = 19^h\ 34^m\ 27^s{\cdot}40$ $\delta = +50^\circ\ 0'\ 4''{\cdot}0$.

By an oversight, reference to a valuable set of spectroscopic observations by Professor Wright of the Lick Observatory was omitted in my previous paper in *M.N.* His paper is in *M.N.*, vol. lxxii. p. 548. The title is *Spectrum of R Cygni.* There is also a paper on the same subject by the Rev. T. E. Espin in the same volume at p. 546.

Yerkes Observatory, Williams Bay, Wisconsin:
1915 *July* 29.

On the Accuracy of Hagen's Chart of T Herculis; and on a possible new Variable Star. By F. A. Bellamy, M.A.

The variable star T Herculis is one of those observed by the late Mr. Baxendell, whose observations are now being edited by Miss Blagg and Professor H. H. Turner. It also falls within the Oxford Astrographic Catalogue, so that it is easy to give accurate positions of the stars in Hagen's Chart (No. 6512 in *Atlas Stellarum Variabilium*).

The Oxford plates available for giving these positions are:—

No.	Centre R.A.	Centre Dec.	Date of Exposure.	No. of Stars.	Ratio to B.D.
	h m	°			
1598	18 4	+31	1900 Sept. 15	621	6·0
1586	18 0	+30	1900 Sept. 12	509	6·6
1587	18 9	+30	1900 Sept. 12	435	4·7

In the Potsdam Astrographic Catalogue, vol. iv. p. 403, plate 121, there are 12 of the stars common to those given in Table I.; but the R.A. ends before T Herculis, and the next plate is unpublished.

The explanation of the material in Table I. is as follows: Columns 1, 2, and 3 give the reference numbers in the Oxford Astrographic Catalogue. The asterisk signifies that the star is in the Leiden A.G.C.

Column 4.—The photographic magnitudes, ϕ, which have been deduced from the Oxford plates by the formula

$$15{\cdot}6 - 1{\cdot}25\sqrt{d}$$

obtained by comparison with the magnitudes for Hagen's grades in *Harvard Annals*, xxxvii. p. 172.

Columns 5 and 6 give the right ascensions and declinations deduced from the corrected Oxford measures—stars north of 31° 5′ depend upon the measures of one plate only,—and the positions are for the epoch 1900·0.

Columns 7 and 8 give Hagen's (A.S.V.) number and grade. It will be seen that a few additional faint stars close to the variable star have been included.

Columns 9 and 10 show the differences $\Delta\alpha$ and $\Delta\delta$ for Hagen-Oxford, given in units of a tenth of a second of time in R.A. and 1″ in Dec.—though the A.S.V. Catalogue only gives the positions to the nearest second of time and tenth of a minute of arc.

TABLE I.

Oxford. No. in Zone. +31°.	+30°.	+30°.	Mag.	R.A. 18h+. m s	N. Dec. ° ′ ″	A.S.V. No.	Grade.	Δα (Unit 0s·1.)	Δδ (Unit 1″.)
41233*	44704*	...	9·1	2 40·87	31 3 54·4	10	38	+ 1	− 6
41329	...	...	9·7	2 46·08	31 27 52·7	21	62	− 1	+ 1
...	44638	...	11·9	2 47·10	30 45 58·3				
41115	44580	...	9·9	2 54·06	30 37 35·0	17	57	− 1	− 17
41184	44658	...	12·3	2 59·46	30 54 54·2				
41205	44676	...	9·7	2 59·84	30 55 43·8	19	60	+ 2	+ 4
...	44677	...	12·1	3 1·29	31 0 8·9				
41185*	44659*	...	8·5	3 7·41	30 54 10·7	8	34	− 4	+ 1
41084*	44547*	...	5·1	3 13·81	30 32 50·6	1	...	+ 2	− 3
41234	44705	...	10·1	3 17·20	31 3 0·9	24	69	− 2	+ 5
41158	44639	...	12·1	3 20·02	30 49 1·8				
...	44706	...	11·9	3 24·29	31 0 51·4				
41206	44678	...	12·1	3 25·16	30 59 18·6				
41254	...	...	11·1	3 30·45	31 7 9·4				
41186	44660	...	12·1	3 31·22	30 52 27·3				
...	44679	...	11·9	3 44·89	30 55 56·0				
...	44680	...	11·5	3 46·93	30 56 8·3				
41207	44681	...	12·8	3 51·24	30 56 49·9				
41235	44707	...	10·3	3 53·85	31 1 19·6	26	74	+ 1	− 8
41159	44640	...	11·3	4 3·36	30 45 35·0	30	81	− 4	− 11
41308	...	...	9·7	4 6 30	31 23 37·2	20	62	− 3	+ 23
41119	44585	..	11·3	4 9·49	30 38 16·2	16	(54)	− 5	+ 2
41236	44708	...	12·3	4 12·29	31 3 55·5	36	97	− 3	+ 10
41160	44641	...	12·5	4 13·31	30 46 29·7	40	105	− 3	− 24
41333	...	...	9·6	4 13·67	31 26 43·1	13	45	− 7	+ 11
41334*	...	...	8·1	4 16·98	31 29 35·6	9	34	− 10	+ 6
41187	44661	...	12·5	4 23·53	30 51 26·3	50	122	− 5	+ 4
41208*	44682*	45199*	6·3	4 25·24	30 59 0·6	4	11	− 2	+ 17
41208*a*	44683	...	13·1	4 25·53	30 56 6·1	42	107	− 5	+ 18
..	44684	...	12·1	4 31·01	30 57 22·2				
41293	...	...	10·0	4 31·48	31 18 35·5	18	58	− 5	+ 6
41336*	...	...	7·8	4 49·16	31 26 35·7	7	25	− 2	0
41262	...	...	12·3	4 49·30	31 10 54·0	44	110	− 3	− 12
41208*b*	...	...	13·4	4 50·11	30 56 15·3	54	141	− 1	+ 9
41209	44685	..	12·5	4 50·36	30 57 6·3	45	113	− 4	+ 6
41337	...	...	9·7	4 53·16	31 26 22·7	12	43	− 2	+ 1

TABLE I.—*continued.*

Oxford. No. in Zone. +31°.	+30°.	+30°.	R.A. Mag.	18h+. m s	N. Dec. ° ′ ″	A.S.V. No.	Grade.	Δα (Unit 0s·1.)	Δδ (Unit 1″.)
41210	44686	45220	11·3	4 56·10	30 59 45·5	31	84	−11	+14
41188	44662	45177	11·9	5 2·64	30 53 52·6	34	93	+ 4	+ 7
41237	44709	45221	11·7	5 15·84	31 1 4·2	29	79	+ 1	− 4
41161	44642	45178	10·4	5 18·94	30 50 21·0	22	65	− 9	−18
41211*	44687*	45222*	Var.	5 19·16	31 0 12·2	...	...	− 2	0
41063*	44524*	45102*	7·0	5 20·85	30 26 52·9	3	0	+ 1	−11
41340	...	...	10·1	5 24·10	31 28 5·4	15	53	− 1	− 5
41263	...	...	11·9	5 31·26	31 13 44·2	27	77	− 3	+16
41142	...	45136	9·5	5 31·62	30 41 17·8	14	47	+ 4	+ 6
41264	...	...	10·6	5 45·25	31 11 34·8	25	71	− 3	+19
41238	...	45223	12·1	5 53·19	31 3 59·2	37	97	− 2	+ 7
41238*a*	...	...	13·4	6 2·05	31 2 14·7	52	131	−11	+21
41265	...	...	12·9	6 9·10	31 12 58·4				
41254*a*	...	...	13·3	6 11·68	31 8 39·8	46	115	+ 3	+14
41266	...	...	11·8	6 13·11	31 15 15·2				
41162*	...	45152*	7·3	6 14·87	30 49 18·3	6	19	+ 1	− 2
41239	...	45224	12·5	6 15·44	31 2 55·8	39	103	− 4	+16
41212	...	45200	12·1	6 21·23	30 57 47·7	43	109	− 2	− 6
41267	...	...	12·1	6 23·64	31 10 33·4	38	100	+ 4	− 3
41294	...	...	12·9	6 29·16	31 16 14·7				
41188*a*	...	...	13·4	6 31·19	30 53 8·8	53	141	−12	+ 9
41313	...	...	10·1	6 35·70	31 21 37·9	11	39	+ 3	+16
41189	...	45179	12·5	6 36·16	30 54 20·0	47	118	− 2	− 8
41255	...	...	11·3	6 36·53	31 10 0·4	33	92	− 5	0
41213	...	45201	12·3	6 39·25	30 56 15·7	51	127	− 3	+ 8
41240	...	45225	11·7	6 50·02	31 4 14·6	32	89	0	− 3
41241	...	45226	10·9	6 54·89	31 3 13·6	28	78	+ 1	− 2
41163	...	45154	12·3	6 59·53	30 45 48·2	49	49	− 5	−12
41164	...	45155	12·1	7 0·72	30 46 25·3	41	105	+ 3	−19
41268*	...	...	8·4	7 11·71	31 13 43·0	5	18	+ 3	− 7
41315	...	...	11·3	7 12·18	31 23 33·0	35	94	− 2	+21
41269	...	...	12·8	7 22·55	31 12 47·7				
41270	...	...	10·8	7 28·95	31 13 34·5	23	68	0	−11
41317*	...	...	6·4	8 8·37	31 22 48·9	2	...	− 4	+17

Considering that, as already mentioned, the A.S.V. positions are only given approximately, the differences in the last two columns are creditably small; and it is noteworthy that they even

show traces of magnitude equation. A summary of the differences is collected in Table II.

TABLE II.

Mean Mag.	Mean $\Delta\alpha$.	Mean $\Delta\delta$.
	s	"
6·9	+·03	+1·8
8·7	−·23	+3·0
9·6	−·27	+0·5
10·2	−·08	−1·2
10·8	−·12	+1·7
11·6	−·28	+8·5
12·1	−·12	−0·8
12·5	−·20	−1·0
13·1	−·58	+6·5
Mean	−·24	+2·2

The magnitude equation in R.A. is indicated by a decided decrease or earlier R.A. between magnitudes 7 and 8 and again below magnitude 12 in the same direction; in each case this direction is contrary to that usually found, for the A.S.V. observations of the fainter stars are too early. In $\Delta\delta$ the residuals are sensibly small and accidental.

It was stated earlier in this paper that some of the positions in Table I. depend upon the mean of two or three plates. It may be of interest to give a brief summary of the separate differences between plates in R.A. and Dec; the subscript figures are the number of stars having that difference:

$^{s}\cdot01_5$ $^{s}\cdot02_5$ $^{s}\cdot03_5$ $^{s}\cdot04_1$ $^{s}\cdot05_6$ $^{s}\cdot06_3$ $^{s}\cdot07_5$ above $^{s}\cdot07_7$, in $\Delta\alpha$.

Less than $0''\cdot2_{13}$ $0''\cdot4_9$ $0''\cdot6_6$ $0''\cdot8_1$ greater than $0''\cdot8_8$, in Dec.

The means are respectively $\pm^{s}\cdot049$ and $\pm0''\cdot42$. For the two pairs of plates we get, taking account of signs:

Mean $\Delta\alpha$ (for $+30°$ 18^h $0^m-31°$) $=+^{s}\cdot026$ and $\Delta\delta=+0''\cdot44$.

Mean $\Delta\alpha$ (for $+30°$ 18^h $9^m-31°$) $=-^{s}\cdot003$ and $\Delta\delta=+0''\cdot16$.

These mean differences would of course be modified by using different plate constants, especially c and f.

The position of T Herculis obtained from the mean of the three plates for 1900·0 is

$$18^h\ 5^m\ 19^s\cdot16 \text{ and } +31°\ 0'\ 12''\cdot2$$

and the photographic magnitude deduced for the epochs 1900 September 12 (J.D. 2415275) and September 15 (J.D. 2415278) are 7·9, 7·6, and 8·0 respectively.

Six of the 54 stars given in the A.S.V. chart were not originally measured on the Oxford plates, so they are not in the printed

volumes. During the progress of this paper the plates were scrutinised in the neighbourhood of these six stars, and the images of the six-minutes' exposure of five of these were detected. The notes made at the time of the scrutiny are, "No. 42, 1st and 2nd images visible, $d=4$; 46, extremely faint, 1st image; 48, nothing visible; 52, extremely faint, 1st exposure only; 53, doubtful image; and 54, 1st image doubtfully visible"; $d=3$ in each case, except No. 42. In four cases the three-minutes' exposure is not visible and the original measure is thus exonerated from any oversight. Evidence is afforded by this scrutiny that there has been no fading of the images on these three plates in nearly fifteen years, since their measurement in 1900 December 17–19 and 1901 March 15.

A.S.V. Nos. 12 and 24 are not included in B.D.; especially should No. 12 have been in that catalogue, as it is 9·1 visual magnitude and photographically it is 9·7. Stars in Oxford zone $+30°$ Nos. 44638, 44677, 44679, 44680, 44684, and 44706 are not in zone $+31°$, though slightly fainter stars are included in Table I. A.S.V. Nos. 14 and 35 there is some suspicion of P.M. or bad positions in B.D. when compared with those places; the available material is too meagre to decide the question.

As to the suspected new variable star. A.S.V. No. 48 has the approximate R.A. $18^h\ 4^m\ 18^s$ and Dec. $+30°\ 57'\ 6''$ (1900·0); as pointed out above, but there is no trace of the star on the Oxford plates. An examination of the Franklin-Adams chart, No. 158, showed that the star is correctly given by Fr. Hagen, and on the F.A. chart it is distinctly brighter than A.S.V. No. 54 close by. This being a direct photographic comparison, the absence of the star from the Oxford plates exposed in 1900 September 1912 and September 15, which show No. 54, cannot be due to the redness of No. 48; it would seem that No. 48 must vary in its light.

University Observatory, Oxford:
1915 *November* 11.

Eighth Note on the Number of Faint Stars with Large Proper Motions. Zone $+29°$. By F. A. Bellamy, M.A.

This note relates to the comparison of two plates in the region covered by the Oxford Astrographic Catalogue with the plate-centre $+29°$, R.A. $12^h\ 31^m$, exposed by me at the two epochs 1896 April 11 (1896·274) and 1911 May 24 (1911·392): both had three exposures of 6^m, 3^m, and 20^s.

The earlier plate, 1000 *b*, was measured by Mr. T. J. Moore in 1898 March and has 179 stars; the later plate, 2866, was measured in 1915 April by Mr. R. Allen, and the measures were

entered by Mr. E. Percy: both being Belgian refugees then resident in Oxford. Plate 2866 has 184 stars.

The actual number compared is 159 stars. Twenty-five stars measured on plate 2866 were not measured on plate 1000 *b*. A scrutiny of plate 1000 *b* showed that for 7 of these stars both 6^m and 3^m images were visible, though the second was usually very faint; for 5 faint stars one of the two images was confused with the réseau line and the star consequently escaped measurement; for 7 stars the second image was invisible; for 3 stars there was considerable doubt about the image being stellar; for 2 stars there was no trace of an image, and 1 star was outside the réseau. A re-examination of these 24 stars on plate 2866 proved that they all showed the two images and were correctly measured.

Twenty-one stars originally measured on plate 1000 *b* were omitted in the measures of plate 2866. The images for both the 6^m and 3^m exposures were visible for 14 of these stars, the second exposure was doubtfully visible for 6 stars, and one was outside the réseau.

The plate constants were derived from a direct comparison of 43 stars to the eleventh magnitude, and the measures of plate 2866, corrected by the deduced formulæ, were compared with the measures of plate 1000 *b* in the usual way. All cases where $(\Delta x)^2+(\Delta y)^2$ was at least 25 (unit = ·001 of a réseau interval of 300″) were marked for special examination, and in Table I. are given those stars for which the suggested motion (equivalent to at least 9″·9 a century) was confirmed.

TABLE I.

Oxford +29°,	ϕ.	R.A. (1900·0).	N. Dec. (1900·0).	Δx.	Δy.	Centennial P.M. in Arc.
		h m s	° ′ ″			″
32914*	10·1	12 26 38·96	29 16 6·1	+·003	−·004	9·9
32907	11·3	26 53·68	29 12 29·7	+ 6	0	11·9
32806	10·5	27 15·63	28 2 5·7	+ 3	+ 4	9·9
32857*	8·6	27 30·29	28 38 25·0	− 1	− 6	12·1
32957	10·5	31 19·86	29 47 6·6	− 5	0	9·9
32810	10·2	31 44·44	28 1 49·0	− 10	− 8	25·4
32879	10·5	34 13·83	28 51 7·8	+ 4	+ 4	11·3
32961*	8·8	34 18·35	29 47 29·9	+ 9	− 10	26·8
32839	11·0	35 6·62	28 21 52·6	− 6	0	11·9

These positions are deduced from the measures of plate 1000 *b*, and are uncorrected for P.M. to epoch 1900·0.

The second column contains the photographic magnitude as deduced by the formula $14{\cdot}2 - 1{\cdot}02\sqrt{d}$ given at the beginning

* These stars are in the Cambridge A.G.C.

of the plate on p. 87 of the Oxford Astrographic Catalogue, vol. iii. I have used the symbol ϕ as an appropriate notation indicating that the magnitudes are photographic and not visual. I am not aware that any definite symbol has hitherto been proposed for this purpose, and possibly the suggestion may be approved generally.

Three of these stars are in the Cambridge A.G.C., and afford a good illustration of the importance of the corrections for magnitude equation given in vol. viii. p. 137 of the Oxford Survey. In the first instance, No. 6152, in Table II. it will be seen that the differences O – C for 13·0 years on (p. 45) are – 6 and 0: these become + 4 and – 2 when corrected for magnitude equation; the Δx and Δy in 15·1 years in Table I. are + 3 and – 4, with which the first pair are quite discordant, whereas the second show good agreement.

TABLE II.

Oxford +29°. No.	Camb. A.G.C. No.	O – C uncorrected for Mag. Eq. Δx.	O – C uncorrected for Mag. Eq. Δy.	Correction for C. Mag. Eq., p. 137. Δx.	Correction for C. Mag. Eq., p. 137. Δy.	Corrected Differences O – C. Δx.	Corrected Differences O – C. Δy.	Interval. Years.	Differences in Table I. Δx.	Differences in Table I. Δy.	Interval. Years.
32914	6152	– 6	0	+10	–2	+ 4	– 2	13·0	+3	– 4	15·1
32857	6158	– 5	– 6	+ 3	–1	– 2	– 7	24·5	–1	– 6	15·1
82961	6194	+12	–12	+ 3	–1	+15	–13	22·6	+9	–10	15·1

It will be seen that when the proper corrections for magnitude equation are applied the two determinations of P.M. become satisfactorily accordant, and may be combined simply by dividing the sums of the differences by the sums of the intervals. The results in centennial P.M. are given in Table III., μ_x and μ_y being expressed in réseau intervals and the resultant μ in seconds of arc.

TABLE III.

Oxford No.	Camb. No.	μ_x.	μ_y.	μ.
32914	6152	+ ·026	– ·021	9″·9
32857	6158	– ·007	– ·033	10·2
32961	6194	+ ·064	– ·061	26·5

University Observatory, Oxford:
1915 *November* 25.

Note on the Nebulæ and Star Clusters shown on the Franklin-Adams Plates. By H. Knox Shaw, M.A.

Two papers have been published recently, one by Mr. Hardcastle in *M.N.*, vol. lxxiv., No. 8, on the nebulæ shown on the Franklin-Adams plates, and the other by Mr. Melotte in *Memoirs*, vol. lx., part v., on the star clusters shown on the same plates. As the plates afford the best available data—in fact the only data on an adequate scale which is uniform over the whole sky,—they will probably be used for some time as a basis for discussion of distribution. In view of this fact it may be of value to have on record in these pages such evidence, whether contradictory or confirmatory of suspicious cases, as is afforded by photographs taken at Helwân during the last seven years, although most of the observations have already been published in *Helwân Observatory Bulletins*, Nos. 9 and 15. The nebulæ and clusters photographed at Helwân lie in the zone between the equator and 45° south declination. The focal length of the Reynolds reflector is three times that of the Franklin-Adams telescope, and thus the plates taken with the former show much more detail in the case of the smaller nebulæ, especially when they appear near the edge of a Franklin-Adams plate where the definition is poor. It is not of much importance if a particular nebula is omitted from the class to which it rightly belongs, but it is important that one should not be included in a class to which it does not belong. Taking the nebulæ first and the classes into which they have been divided by Mr. Hardcastle in their order, I have the following comments to make.

NEBULÆ.

Class I. (a).—Spirals.

N.G.C. 246 is not a spiral, but like the Owl nebula.

Class I. (b).—Probably Spirals.

N.G.C. 55, 134, 150, 1518, 1532, 1808, 2207, 2280, 2835, 3511, 3621, 4027, 4487, 5427, 6907, and 7793 are shown as spirals on Helwân photographs, and N.G.C. 157, 894–5, 1784, and 7184 are given as spirals in *Lick Observatory Bulletin*, No. 219. I.C. 2132, included in this list, is probably a mistake for N.G.C. 1954, which is a spiral. I.C. 2132 is a spindle E 0°.

Class I. (c).—Known to be Spirals.

N.G.C. 246, see under I. (*a*).
5897 is a globular cluster.
7293 is not a spiral; it is also included in Class III., where it properly belongs.

Class II.—Spindle-shaped Nebulæ.

Many of these have been photographed at Helwân and classed as spindles or possible spirals. Of the following it can definitely be said that

N.G.C. 253 is a large well-known spiral.
289 is an open spiral.
1360 is not a spindle; it is $6' \times 3'$, and probably like the Owl.
3309 is round and is probably included by mistake for one of the many other small nebulæ near.
4038 is an open spiral.
4304 is a circular spiral.
4602 is an elongated spiral.
5507 is probably a mistake for 5506, which is much elongated; 5507 is round.
5728 is an elongated spiral.
6293 is a globular cluster, and not elongated.
6925 is an elongated spiral.
7721 is an elongated spiral.

N.G.C. 1084, 5483, and 5595 have been photographed at Lick, Harvard, and Johannesburg respectively, and announced as spirals.

Class III.—Diffuse Nebulæ.

N.G.C. 2517 is a very small and well-defined nebula. N.G.C. 3672 and 7314 are spirals.

Class IV.—Small Nebulæ.

The following are spirals: N.G.C. 1309, 3513, 3887, 4902, and 7555. N.G.C. 6235 is a globular cluster, and so probably is 6355, but only a visual observation of this latter has been made.

A certain number of fairly large nebulæ, *i.e.* more than one minute in diameter, which have been photographed at Helwân, are not included in Mr. Hardcastle's lists. Some of these I have been able to identify without much difficulty on the charts printed photographically from these plates and published by the Society. Others, no doubt, would have been shown had they not been near the edge of a plate. Thus the lists cannot be taken as being complete down to a certain size and brightness; in fact Mr. Hardcastle does not claim that they are, as he describes his lists as being composed of the more conspicuous objects shown on the plates.

It is of interest to see how these further observations modify the diagrams which Mr. Hinks added to Mr. Hardcastle's paper. Let us consider square A, the galactic square shunned by spirals and spindles. In fig. 1, Plate 18, there is one nebula, N.G.C. 6907, taken from Class I. (*b*); this is confirmed as a spiral. In fig. 2 the object shown near the galaxy, N.G.C. 6293, is a globular cluster; and of the other three objects south of the galactic plane,

one, N.G.C. 6925, is a spiral, and the other two, I.C. 5039 and 5041, have been observed visually at Helwân and are apparently very faint spindles. In fig. 3 of the five objects near the galaxy I have been unable to identify three with nebulæ given in Class IV. Of the other two N.G.C. 6235 is certainly, and 6355 is probably, a globular cluster. Thus the absence of "white" nebulæ from this square becomes even more pronounced.

Star Clusters.

Amongst those noted by Professor Bailey, but not included in Mr. Melotte's catalogue, the Helwân plates confirm N.G.C. 1291 as a nebula, and 1380 and 1399 are also almost certainly so.

Amongst the objects classed as globular clusters in the *N.G.C.*, and not identified by Mr. Melotte,

N.G.C. 1310 is probably not a globular cluster, but a nebula.
1379 and 1387 are similar to 1380 and 1399, and probably nebulæ.
1436 does not exist.
4027 is an open spiral, classed by Hardcastle as probably spiral.
6629 is a planetary nebula.
7214 is an open spiral.

N.G.C. 6884 and 6891 are meant to be classed as planetary nebulæ in the *N.G.C.*, but the type is bad.

Of the objects included in the catalogue N.G.C. 2818 is a small gaseous nebula, not quite planetary, in a cluster. It is given as a planetary nebula in the *N.G.C.*, but the type is bad, as in the case above.

Helwân Observatory:
1915 *August.*

The Distribution of Stars in Globular Clusters.

By H. C. Plummer, M.A.

1. Some valuable statistical material for the study of the distribution of stars in clusters has recently been published by Professor S. I. Bailey.* This includes very careful counts on ten different clusters. From the counts the apparent density, or number of stars per square minute of arc, is deduced for different distances from the centre of the cluster. It is proposed to examine the data provided by Professor Bailey in the light of the ideas developed in a former paper.† A general account of these ideas and their later development has been given elsewhere.‡

* *H.C.O. Annals*, vol. lxxvi. No. 4.
† *Monthly Notices*, lxxi. p. 460.
‡ *Nature*, vol. xciv. p. 674.

It is supposed that the arrangement of the stars in a cluster follows a law analogous to the law of density in a spherical mass of gas in convective equilibrium. This law is expressed by the equation

$$\frac{d^2(r\rho^{\gamma-1})}{dr^2} + r\rho = 0 \quad . \quad . \quad . \quad . \quad (1)$$

where ρ is the density at the distance r from the centre and γ is the ratio of the specific heats at constant pressure and constant volume. One particular solution is known which takes a simple algebraic form and corresponds to the value $\gamma = 1{\cdot}2$. In terms suitable for the present problem this may be written

$$\rho = \phi(r) = \frac{3}{4}\frac{N}{a}\left(1 + \frac{r^2}{a^2}\right)^{-2{\cdot}5} \quad . \quad . \quad . \quad (2)$$

It is easily deduced that the corresponding apparent density projected on a plane, as in a photograph, is

$$f(r) = N\left(1 + \frac{r^2}{a^2}\right)^{-2} \quad . \quad . \quad . \quad . \quad (3)$$

so that N is the central density or number of stars in unit area at the centre of the cluster. The distribution is infinite, with no finite boundary. But the total number of stars is finite, and is in fact

$$N_\infty = \pi N a^2 \quad . \quad . \quad . \quad . \quad (4)$$

The constant a may be regarded as the radius of the equivalent homogeneous or uniform cluster, containing the same number of stars and having the same volume density, or the same areal density, as the actual cluster at its centre.

2. The above law was found to be followed with remarkable accuracy by the cluster ω Centauri, the value of a being $7'{\cdot}14$. In other cases the law has been applied with different degrees of success, and certain modifications have been suggested without departing from the same fundamental order of ideas. It is therefore convenient to compare the new material with the simple law expressed by (3) as a standard. The systematic comparison can be conducted in the following way. Let

$$n = f(r) = N\left(1 + \frac{r^2}{a^2}\right)^{-2}.$$

Then

$$n^{-\frac{1}{2}} = N^{-\frac{1}{2}}\left(1 + \frac{r^2}{a^2}\right).$$

The given numbers n and the derived numbers $n^{-\frac{1}{2}}$ are tabulated with $30''$ as the common difference of the argument r. It is convenient to adopt this as the unit to which r is referred. The result of differencing the table is to give

$$\Delta n^{-\frac{1}{2}} = (2r - 1)N^{-\frac{1}{2}}/a^2,$$

and hence

$$\tfrac{1}{2}\Delta^2 n^{-\frac{1}{2}} = N^{-\frac{1}{2}}/a^2 = \Delta n^{-\frac{1}{2}}/(2r - 1).$$

The meaning of this is that when the first differences of $n^{-\frac{1}{2}}$ are divided by $(2r-1)$ a constant should result. A direct test of the theory is thus afforded without any preliminary knowledge of N and a. In practice an absolutely constant value is scarcely to be expected, but the numbers will suggest a suitable mean value. The process may now be reversed, and if m is the theoretical number corresponding to n,

$$\Delta m^{-\frac{1}{2}} = (2r-1)\mathrm{N}^{-\frac{1}{2}}/a^2.$$

If any one value of $m^{-\frac{1}{2}}$ be now assumed, the whole sequence of values can be constructed successively by the aid of these first differences. This done, the values of m follow immediately and the theoretical numbers m can be compared with the observed numbers n. One remark is necessary. The unit here adopted is 0′·5, whereas the densities are referred to the unit 1′. Hence the total number of stars in a cluster, when calculated by (4), will be multiplied by 4 and must be reduced by this factor to be made comparable with the actual count.

3. Table I. gives the result of comparing the densities derived for the cluster N.G.C. 362 with the theoretical distribution,

Table I. N.G.C. 362.

r.	n.	$n^{-\frac{1}{2}}$.	$\Delta n^{-\frac{1}{2}}$.	$\frac{1}{2}\Delta^2 n^{-\frac{1}{2}}$.	$\Delta m^{-\frac{1}{2}}$.	$m^{-\frac{1}{2}}$.	m.	$n-m$.
0	(224·8)	...	...	...	...	·086	135·3	(+89·5)
1	110·5	·0952	...	...	·009	·095	110·9	− 0·4
2	63·0	·1259	·0307	·0102	·027	·122	67·2	− 4·2
3	36·5	·1656	·0397	·0079	·045	·167	35·9	+ 0·6
4	21·5	·2155	·0499	·0071	·063	·230	18·9	+ 2·6
5	11·5	·2950	·0795	·0088	·081	·311	10·4	+ 1·1
6	6·0	·4082	·1132	·0103	·099	·410	6·0	0
7	3·6	·5263	·1181	·0091	·117	·527	3·6	0
8	2·7	·6098	·0835	·0056	·135	·662	2·3	+ 0·4
9	1·9	·7246	·1148	·0068	·153	·815	1·5	+ 0·4
10	1·3	·8772	·1526	·0080	·171	·986	1·0	+ 0·3
11	0·9	...	...	...	·189	1·175	0·72	+ 0·18
12	0·7	...	...	...	·207	1·382	0·52	+ 0·18
13	0·5	...	...	...	·225	1·607	0·38	+ 0·12
14	0·4	...	...	...	·243	1·850	0·29	+ 0·11
15	0·3	...	...	...	·261	2·111	0·22	+ 0·08
16	0·2	...	...	...	·279	2·390	0·18	+ 0·02
17	0·2	...	...	...	·297	2·687	0·14	+ 0·06
18	0·2	...	...	...	·315	3·002	0·11	+ 0·09
19	0·1	...	...	...	·333	3·335	0·09	+ 0·01
20	0·1	...	...	...	·351	3·686	0·07	+ 0·03

according to the method explained above. The fifth column is derived from the fourth by division, not by differencing, because

the latter process is clearly rendered illusory by unavoidable irregularities. In adopting the mean value ·009 greater weight is given to the earlier numbers in the fifth column, since these should be the more accurate, and for this reason the column has not been completed. The seventh column is derived from the sixth by accepting the value of $n^{-\frac{1}{2}}$ for $r=1$ from the third column.

The constants which are given by this comparison are

$$N = 135{\cdot}3, \qquad N^{-\frac{1}{2}}a^{-2} = {\cdot}009,$$

which lead to

$$a = 3{\cdot}09 = 93'', \qquad N_\infty = \tfrac{1}{4}\pi N a^2 = 1015.$$

The total number actually counted in the cluster, less the allowance made for the background, is 1058. "The count was made with special care . . . The central 4 squares were estimated." Since a density of 144 gives one star to every square of side 5″ with regular spacing, it is obvious that the estimate of the central density is difficult. It must, however, be taken as it stands as evidence that in the immediate vicinity of the centre the condensation is sensibly in excess. With this exception the agreement with the theory seems eminently satisfactory.

4. Table II. gives a similar comparison for the cluster N.G.C.

Table II. N.G.C. 5272.

r.	n.	$n^{-\frac{1}{2}}$.	$\Delta n^{-\frac{1}{2}}$.	$\frac{1}{2}\Delta^2 n^{-\frac{1}{2}}$.	$\Delta m^{-\frac{1}{2}}$.	$m_{-\frac{1}{2}}$.	m.	$n-m$.
0	(143·4)	...	...	...	...	·100	100	(+43 4)
1	104·0	·0980	...	....	·007	·107	87·4	+16·6
2	62·0	·1271	·0291	·0097	·021	·128	61·0	+ 1·0
3	37·5	·1634	·0363	73	·035	·163	37·6	− 0·1
4	23·0	·2083	·0449	64	·049	·212	22·3	+ 0·7
5	14·0	·2674	·0591	66	·063	·275	13·3	+ 0·7
6	8·0	·3534	·0860	78	·077	·352	8·1	− 0·1
7	4·6	·4673	·1139	88	·091	·443	5·1	− 0·5
8	3·3	·5495	·0822	55	·105	·548	3·3	0
9	2·5	·6329	·0834	49	·119	·667	2·3	+ 0·2
10	2·0	·7092	·0763	40	·133	·800	1·3	+ 0·7
11	1·7	·7692	·0600	29	·147	·947	1·1	+ 0·6
12	1·4	·8475	·0723	31	·161	1·108	0·8	+ 0·6
13	1·2	·9091	·0616	25	·175	1·283	0·6	+ 0·6
14	1·0	...	·0909	·0034	·189	1·472	0·5	+ 0·5
15	0·8	...	...	...	·203	1·675	0·4	+ 0·4
16	0·6	...	...	...	·217	1·892	0·3	+ 0·3
17	0·5	...	...	...	·231	2·123	0·2	+ 0·3
18	0·4	...	...	...	·245	2·368	0·2	+ 0·2
19	0·3	...	...	...	·259	2·627	0·2	+ 0·1
20	0·2	...	...	...	·273	2·900	0·1	+ 0·1

5272, otherwise known as M 3. "This count was made with special care. . . . The central 4 squares were estimated." From the table

$$N^{-\frac{1}{2}}a^{-2} = \cdot 007, \qquad N = 100,$$

and hence

$$a = 3\cdot 77 = 113'', \qquad N_\infty = 1122,$$

while the total number of stars counted in the cluster was 1192. The tolerable appearance of the residuals scarcely bears examination, since towards the end of the table the calculated densities are systematically half the observed values. The outer densities require the mean value of the fifth column to be ·004 at most, but such a value will not represent the inner densities. There is an essential discrepancy between the inner and outer parts of the cluster, which is shown clearly by the run of the numbers in the fifth column. Much study has been given to this cluster, and it was known that its distribution was inconsistent with the simple law which has been taken as the standard of comparison.

5. Table III. gives the comparison of the cluster N.G.C. 5986 with the theory. Here

$$N^{-\frac{1}{2}}a^{-2} = \cdot 019, \qquad N = 135\cdot 3,$$

whence

$$a = 2\cdot 13 = 64'', \qquad N_\infty = 481.$$

The actual count assigns 460 stars to the cluster. The residuals are quite satisfactory, and leave no doubt that this cluster follows the simple law of distribution.

TABLE III. N.G.C. 5986.

r.	n.	$n^{-\frac{1}{2}}$.	$\Delta n^{-\frac{1}{2}}$.	$\frac{1}{2}\Delta^2 n^{-\frac{1}{2}}$.	$\Delta m^{-\frac{1}{2}}$.	$m^{-\frac{1}{2}}$.	m.	$n-m$.
0	134·6	...	...	...	...	·086	135·3	−0·7
1	88·0	·1066	...	...	·019	·105	90·6	−2·6
2	41·0	·1563	·0497	·0166	·057	·162	38·1	+2·9
3	17·0	·2427	·0864	·0173	·095	·257	15·1	+1·9
4	6·6	·3891	·1464	·0209	·133	·390	6·6	0
5	3·4	·5435	·1544	·0172	·171	·561	3·2	+0·2
6	1·8	·7463	·2028	·0184	·209	·770	1·7	+0·1
7	1·0	...	·2537	·0195	·247	1·017	0·97	+0·03
8	0·6	...	...	...	·285	1·302	0·59	+0·01
9	0·3	...	...	...	·323	1·625	0·38	−0·08
10	0·2	...	...	...	·361	1·986	0·25	−0·05

6. Table IV. gives the comparison between the standard distribution and the densities observed in the cluster N.G.C. 6205 or M 13, the great cluster in Hercules. In this case

$$N^{-\frac{1}{2}}a^{-2} = \cdot 004, \qquad N = 110\cdot 9,$$

and accordingly

$$a = 4\cdot 88 = 146'', \qquad N_\infty = 2074,$$

whereas the complete count gives 2146 stars. But for an excessive density between 2′ and 3′·5 from the centre of the cluster the distribution seems to be very fairly represented.

TABLE IV. N.G.C. 6205.

r.	n.	$n^{-\frac{1}{2}}$.	$\Delta n^{-\frac{1}{2}}$.	$\frac{1}{2}\Delta^2 n^{-\frac{1}{2}}$.	$\Delta m^{-\frac{1}{2}}$.	$m^{-\frac{1}{2}}$.	m.	$n-m$.
0	132·6	...	...	...	...	·095	110·9	+21·7
1	110·0	·0953	...	..	·004	·099	102·0	+ 8·0
2	76·0	·1147	·0194	·0065	·012	·111	81·2	− 5·2
3	58·5	·1307	·0160	32	·020	·131	58·2	+ 0·3
4	45·5	·1481	·0174	25	·028	·159	39·6	+ 5·9
5	33·5	·1727	·0246	27	·036	·195	26·3	+ 7·2
6	23·5	·2062	·0335	30	·044	·239	17·6	+ 5·9
7	15·5	·2538	·0476	37	·052	·291	11·8	+ 3·7
8	9·0	·3333	·0795	53	·060	·351	8·1	+ 0·9
9	5·6	·4219	·0886	52	·068	·419	5·7	− 0·1
10	3·9	·5076	·0857	45	·076	·495	4·1	− 0·2
11	3·0	·5780	·0704	34	·084	·579	3·0	0
12	2·3	·6579	·0799	35	·092	·671	2·2	+ 0·1
13	1·8	·7463	·0884	35	·100	·771	1·7	+ 0·1
14	1·4	·8475	·1012	37	·108	·879	1·3	+ 0·1
15	1·0	...	·1525	·0053	·116	·995	1·0	0
16	0·8	...	...	...	·124	1·119	0·8	0
17	0·6	...	...	...	·132	1·251	0·6	0
18	0·4	...	...	...	·140	1·391	0·5	− 0·1
19	0·2	...	...	...	·148	1·539	0·4	− 0·2
20	0·2	...	...	...	·156	1·695	0·3	− 0·1

7. Table V. shows the results of the comparison for the cluster N.G.C. 6266 or M 62. The table gives

$$N^{-\frac{1}{2}}a^{-2} = \cdot 006, \qquad N = 90\cdot 6,$$

whence

$$a = 4\cdot 18 = 125'', \qquad N_\infty = 1246.$$

The whole number of stars counted was 1088, which may seem considerably in defect. The discrepancy is, however, explained by the limit to which the counts were carried. The number of stars belonging to the cluster beyond the radius r is

$$N_\infty \Big/ \left(1 + \frac{r^2}{a^2}\right).$$

In the present case r/a is about 2·63, and hence the theoretical number of stars outside the radius 330″ is 157. The discrepancy therefore disappears. "The number of stars in the central 4 squares was estimated. The cluster is very unsymmetrical." To this point further reference will be made. The distribution is represented very tolerably beyond 1′ from the centre, but

within this distance there is a marked excess of stars beyond the number allowed by the theory. A common tendency is shown by the run of the fifth column of the table, where the numbers first fall to a minimum and then increase.

TABLE V. N.G.C. 6266.

r.	n.	$n^{-\frac{1}{2}}$.	$\Delta n^{-\frac{1}{2}}$.	$\frac{1}{2}\Delta^2 n^{-\frac{1}{2}}$.	$\Delta m^{-\frac{1}{2}}$.	$m^{-\frac{1}{2}}$.	m.	$n-m$.
0	(194·6)	...	...	...	...	·105	90·6	(+104·0)
1	117·0	·0923	...	...	·006	·111	81·2	+ 35·8
2	60·0	·1290	·0367	·0122	·018	·129	60·1	− 0·1
3	36·5	·1656	·0366	·0073	·030	·159	39·6	− 3·1
4	22·5	·2110	·0454	·0065	·042	·201	24·7	− 2·2
5	13·7	·2703	·0593	·0066	·054	·255	15·4	− 1·7
6	9·6	·3226	·0523	·0048	·066	·321	9·7	− 0·1
7	7·1	·3759	·0533	·0041	·078	·399	6·3	+ 0·8
8	5·0	·4464	·0705	·0047	·090	·489	4·2	+ 0·8
9	3·4	·5435	·0971	·0057	·102	·591	2·9	+ 0·5
10	2·3	·6579	·1144	·0060	·114	·705	2·0	+ 0·3
11	1·5	·8197	·1618	·0077	·126	·831	1·4	+ 0·1

8. Table VI. presents the result of applying the theory to the cluster N.G.C. 6809 or M 55. "This is an open cluster . . .

TABLE VI. N.G.C. 6809.

r.	n.	$n^{-\frac{1}{2}}$.	$\Delta n^{-\frac{1}{2}}$.	$\frac{1}{2}\Delta^2 n^{-\frac{1}{2}}$.	$\Delta m^{-\frac{1}{2}}$.	$m^{-\frac{1}{2}}$.	m.	$n-m$.
0	56·0	...	...	...	...	·134	55·7	+0·3
1	54·5	·1355	...	...	·005	·139	51·7	+2·8
2	41·0	·1563	·0208	·0069	·015	·154	42·1	−1·1
3	30·0	·1825	·0262	52	·025	·179	31·2	−1·2
4	21·5	·2155	·0330	47	·035	·214	21·8	−0·3
5	15·0	·2584	·0429	48	·045	·259	14·9	+0·1
6	9·8	·3195	·0611	56	·055	·314	10·1	−0·3
7	6·4	·3953	·0758	58	·065	·379	7·0	−0·6
8	4·8	·4566	·0613	41	·075	·454	4·8	0
9	3·7	·5208	·0642	38	·085	·539	3·4	+0·3
10	2·9	·5882	·0674	35	·095	·634	2·5	+0·4
11	2·3	·6579	·0697	33	·105	·739	1·8	+0·5
12	1·7	·7692	·1113	48	·115	·854	1·4	+0·3
13	1·3	·8772	·1080	·0043	·125	979	1·0	+0·3
14	0·9	...	...	...	·135	1·114	0·80	+0·10
15	0·6	...	...	...	·145	1·259	0·63	−0·03
16	0·5	...	...	...	·155	1·414	0·50	0
17	0·4	...	...	...	·165	1·579	0·40	0
18	0·3	...	...	...	·175	1·754	0·32	−0·02
19	0·3	...	...	...	·185	1·939	0·27	+0·03
20	0·2	...	...	...	·195	2·134	0·22	−0·02

so that the stars could be counted readily even near the centre." According to the table

$$N^{-\frac{1}{2}}a^{-2} = \cdot 005, \qquad N = 55\cdot 7,$$

whence it follows that

$$a = 5\cdot 18 = 155'', \qquad N_\infty = 1172,$$

the number of stars actually counted being 1139. The residuals must be considered entirely satisfactory, and the cluster is a good example of what is here taken to be the standard form.

9. Table VII. gives the comparison for the cluster N.G.C. 7078 or M 15. "The numbers for the central 4 squares are estimates." In this case

$$N^{-\frac{1}{2}}a^{-2} = \cdot 0085, \qquad N = 100\cdot 4,$$

and therefore

$$a = 3\cdot 43 = 103'', \qquad N_\infty = 926.$$

The count gave 1053 stars, and the discrepancy can evidently be attributed to the marked excess of stars at the centre of the cluster. Except within 1′ of the centre the representation is excellent.

TABLE VII. N.G.C. 7078.

r.	n.	$n^{-\frac{1}{2}}$.	$\Delta n^{-\frac{1}{2}}$.	$\frac{1}{2}\Delta^2 n^{-\frac{1}{2}}$.	$\Delta m^{-\frac{1}{2}}$.	$m^{-\frac{1}{2}}$.	m.	$n-m$.
0	(307·8)	...	...	...	...	·0998	100·4	(+207·4)
1	148·0	·0822	...	...	·0085	·1083	85·2	+ 62·8
2	57·0	·1325	·0503	·0168	·0255	·1338	55·8	+ 1·2
3	33·0	·1742	·0417	·0083	·0425	·1763	32·2	+ 0·8
4	18·0	·2358	·0616	·0088	·0595	·2358	18·0	0
5	10·5	·3086	·0728	·0081	·0765	·3123	10·2	+ 0·3
6	5·5	·4255	·1169	·0106	·0935	·4058	6·1	− 0·6
7	3·0	·5780	·1525	·0117	·1105	·5163	3·8	− 0·8
8	2·0	·7092	·1312	·0087	·1275	·6438	2·4	− 0·4
9	1·5	·8197	·1105	·0065	·1445	·7883	1·6	− 0·1
10	1·1	·9524	·1327	·0070	·1615	·9498	1·1	0
11	0·8	...	...	...	·1785	1·1283	0·79	+ 0·01
12	0·6	...	...	...	·1955	1·3238	0·58	+ 0·02
13	0·5	...	...	...	·2125	1·5363	0·42	+ 0·08
14	0·4	...	...	...	·2295	1·7658	0·32	+ 0·08
15	0·3	...	...	...	·2465	2·0123	0·25	+ 0·05
16	0·2	...	...	...	·2635	2·2758	0·19	+ 0·01
17	0·2	...	...	...	·2805	2·5563	0·15	+ 0·05
18	0·2	...	...	...	·2975	2·8538	0·12	+ 0·08
19	0·1	...	...	...	·3145	3·1683	0·10	0
20	0·1	...	...	...	·3315	3·4998	0·08	+ 0·02

10. Table VIII. shows the comparison of the cluster N.G.C. 7089 (M 2) with the theory. According to the table

$$N^{-\frac{1}{2}}a^{-2} = \cdot 009, \qquad N = 170\cdot 8,$$

and therefore

$$a = 2\cdot 91 = 87'', \qquad N_\infty = 1140,$$

the total by actual count being 1133. "The numbers for the central 4 squares are estimates," and exceed the central density according to the assumed law. For the rest the representation is good, though the cluster appears to die out too suddenly at the limit to which the counts were carried.

TABLE VIII. N.G.C. 7089.

$r.$	$n.$	$n^{\frac{1}{2}}.$	$\Delta n^{-\frac{1}{2}}.$	$\frac{1}{2}\Delta^2 n^{-\frac{1}{2}}.$	$\Delta m^{-\frac{1}{2}}.$	$m^{-\frac{1}{2}}.$	$m.$	$n-m.$
0	(269·8)	...	...	...	...	·0765	170·8	(+99·0)
1	142·0	·0839	...	...	·009	·0855	136·9	+ 5·1
2	79·0	·1125	·0286	·0095	·027	·1125	79·0	0
3	40·0	·1582	·0457	91	·045	·1575	40·3	− 0·3
4	20·5	·2208	·0626	89	·063	·2205	20·6	− 0·1
5	10·5	·3086	·0878	98	·081	·3015	11·0	− 0·5
6	6·0	·4082	·0996	91	·099	·4005	6·2	− 0·2
7	3·7	·5208	·1126	87	·117	·5175	3·7	0
8	2·4	·6452	·1244	83	·135	·6525	2·3	+ 0·1
9	1·7	·7692	·1240	73	·153	·8055	1·5	+ 0·2
10	1·1	·9524	·1832	·0096	·171	·9765	1·0	+ 0·1
1	0·8	...	...	...	·189	1·1655	0·74	+0·06
12	0·5	...	...	...	·207	1·3725	0·53	−0·03
13	0·3	...	...	...	·225	1·5975	0·40	−0·10
14	0·2	...	...	...	·243	1·8405	0·29	−0·09
15	0·1	...	...	...	·261	2·1015	0·23	−0·13
16	0·0	...	...	...	·279	2·3805	0·18	−0·18

11. Table IX. gives the representation of the cluster N.G.C. 7099 (M 30) by the same law. In this case

$$N^{-\frac{1}{2}}a^{-2} = \cdot 914, \qquad N = 61\cdot 0,$$

and therefore

$$a = 3\cdot 02 = 91'', \qquad N_\infty = 438,$$

while the count gave a total of 466 stars belonging to the cluster. "The number of stars in a part of the central 4 squares was estimated." Here again, as in some other clusters, the central density is excessive. But except within 1′ of the centre the distribution is in excellent agreement with the law.

TABLE IX. N.G.C. 7099.

r.	n.	$n^{-\frac{1}{4}}$.	$\Delta n^{-\frac{1}{4}}$.	$\frac{1}{2}\Delta^2 n^{-\frac{1}{4}}$.	$\Delta m^{-\frac{1}{4}}$.	$m^{-\frac{1}{4}}$.	m.	$n-m$.
0	(121·4)	...	...	...	...	·128	61·0	(+60·4)
·1	66·0	·1232	...	...	·014	·142	49·6	+16·4
2	29·5	·1842	·0610	·0203	·042	·184	29·5	0
3	15·5	·2538	·0696	·0139	·070	·254	15·5	0
4	8·0	·3534	·0996	·0142	·098	·352	8·1	− 0·1
5	4·5	·4717	·1183	·0131	·126	·478	4·4	+ 0·1
6	2·5	·6329	·1612	·0147	·154	·632	2·5	0
7	1·6	·7937	·1608	·0124	·182	·814	1·5	+ 0·1
8	1·0	...	·2063	·0138	·210	1·024	0·96	+ 0·04
9	0·7	...	...	...	·238	1·262	0·62	+ 0·08
10	0·6	...	...	...	·266	1·528	0·42	+ 0·18
11	0·4	...	...	...	·294	1·822	0·30	+ 0·10
12	0·3	...	...	...	·322	2·144	0·22	+ 0·08
13	0·2	...	...	...	·350	2·494	0·16	+ 0·04
14	0·2	...	...	...	·378	2·872	0·12	+ 0·08
15	0·1	...	...	...	·406	3·278	0·09	+ 0·01
16	0·1	...	...	...	·434	3·712	0·07	+ 0·03

12. It will be evident that the accurate determination of the curve of densities in a cluster is attended with very considerable difficulties on the practical side, although such difficulties have been surmounted with much success in the present series of counts by the exercise of great skill and care. At the centre of a cluster the density is generally so great that the process of counting becomes impossible and it is necessary to substitute more or less trustworthy estimates. In all cases there are doubtless fainter stars in vast numbers which do not enter into the counts at all. An exhaustive count is out of the question: all that is possible is to count the stars down to a definite order of magnitude. This suggests a source of error in the central estimates. For in the less dense regions the fainter stars occur singly and impress no image on the plate. But in the crowded regions two or more faint stars may fall so close to one another as to produce the appearance of one brighter star, thus tending to exaggerate the estimate. There is a difficulty of another kind in the outer regions of a cluster, where much depends on the accurate estimate of the general density of the sky which forms the background to the cluster. If this density be estimated too close to the cluster, it will be influenced by the cluster itself. If it be gauged at a considerable distance from the cluster, local variations in the natural density of the sky may render the estimate inappropriate. It must be added that there is little to suggest that this particular difficulty has had any serious effect, the necessary correction being generally very small.

The stars were counted within small squares and the deduced

densities were plotted according to the distances of the squares from the centre of the cluster. A smooth curve was drawn through the points obtained in this way, and the numbers n in any one of the above tables were read from such a curve at intervals of 30″, which is here taken as the unit of r. They are therefore the result of a graphical interpolation. This is probably the most satisfactory process available, and it has been applied with skill and success. But it is well to bear in mind that the numbers n are not quite raw material, and it is not necessary to attach any great weight to small deviations from the theoretical curve even when they present a systematic run. For this is to be expected when comparison is made, not with individual observations, but with a smoothed curve.

TABLE X. N.G.C. 104.

r.	n.	$n^{-\frac{1}{4}}$.	$\Delta n^{-\frac{1}{4}}$.	$\frac{1}{2}\Delta^2 n^{-\frac{1}{4}}$.	$\Delta m^{-\frac{1}{4}}$.	$m^{-\frac{1}{4}}$.	m.	$n-m$.
0	(139·8)	...	...	...	...	·134	55·7	(+84·1)
1	112·0	·0945	...	...	·001	·135	54·9	+57·1
2	71·5	·1182	·0237	·0079	·003	·138	52·6	+18·9
3	55·0	·1348	·0166	33	·005	·143	48·9	+ 6·1
4	45·0	·1490	·0142	20	·007	·150	44·6	+ 0·4
5	38·0	·1623	·0133	15	·009	·159	39·6	− 1·6
6	32·5	·1754	·0131	12	·011	·170	34·6	− 2·1
7	29·0	·1855	·0101	08	·013	·183	29·8	− 0·8
8	25·5	·1980	·0125	08	·015	·198	25·5	0
9	21·5	·2155	·0175	10	·017	·215	21·6	− 0·1
10	18·5	·2326	·0171	09	·019	·234	18·2	+ 0·3
11	16·0	·2500	·0174	08	·021	·255	15·4	+ 0·6
12	13·2	·2755	·0255	11	·023	·278	13·0	+ 0·2
13	11·0	·3012	·0257	10	·025	·303	10·9	+ 0·1
14	9·2	·3300	·0288	11	·027	·330	9·2	0
15	7·9	·3559	·0259	09	·029	·359	7·8	+ 0·1
16	6·8	·3831	·0272	09	·031	·390	6·6	+ 0·2
17	5·8	·4149	·0318	10	·033	·423	5·6	+ 0·2
18	4·9	·4525	·0376	11	·035	·458	4·8	+ 0·1
19	4·1	·4950	·0425	11	·037	·495	4·1	0
20	3·3	·5495	·0545	·0014	·039	·534	3·5	− 0·2
vi	3·3	...	...	...	·121	·534	3·5	− 0·2
vii	2·1	...	...	...	·143	·677	2·2	− 0·1
viii	1·4	...	...	...	·165	·842	1·4	0
ix	0·9	...	...	...	·187	1·029	0·9	0

13. Special attention was devoted by Professor Bailey to the cluster N.G.C. 104 or 47 Toucanæ, after ω Centauri the finest

cluster in the sky. Four separate counts were made on two plates taken with different instruments. It must be admitted, after an inspection of the results, that these counts illustrate the difficulties and uncertainties of the task rather than combine to throw light on the structure of the cluster. In count (*a*), made on the first plate, all suspected objects were included. In count (*b*), made on the same plate, "all doubtful objects were omitted." Hence the curve corresponding to (*b*) should be everywhere below the curve corresponding to (*a*). But on the contrary, the curve (*b*) is at first above the curve (*a*), crosses it at 3′ from the centre, and then remains below as it should. This anomalous result is the effect of graphical interpolation applied to independent counts which for the most part are nearly equal.

Counts (*c*) and (*d*) were also made on one plate, (*c*) of stars brighter than a selected pair, (*d*) of stars fainter than the same pair. The latter only begins at 5′ from the centre. Near this point the sum of (*c*) and (*d*) gives a density nearly equal to (*a*). According to this indication the exposures on the two plates should be nearly equivalent. But at greater distances from the centre the combined densities corresponding to (*c*) and (*d*) tend to be about double the density shown by (*a*). It is difficult to understand this feature, unless the minimum magnitude recorded on one or both plates is a function of the star's position on the plate. Whatever the cause, the results from the two plates are not consistent, and the differences between them are scarcely relevant to the present purpose. It is enough to notice this possible source of error when the counts extend over a considerable area on the plate.

14. In order to have a comparison of the densities observed in the cluster N.G.C. 104 with the standard law it will suffice to adopt the system of the curve (*a*). Table X. is therefore based on this system of counts. Here

$$N^{-\frac{1}{2}}a^{-2} = \cdot 001, \qquad N = 55\cdot 7,$$

and hence

$$a = 11\cdot 57 = 347'', \qquad N_\infty = 5859,$$

The cluster is by no means confined within the radius 900″ to which these counts extend. At this point $r/a = 2\cdot 59$, and beyond this limit there should be 751 stars, leaving 5108 stars within it. This compares fairly with the number 4942 actually counted. Up to 600″ the unit of r is 30″, as in the previous tables. At this point it is changed to 100″, as indicated by the Roman numerals. The effect of this change of units is to multiply the mean value of the fifth column by $(100/30)^2$, so that it becomes ·011, and the calculation is completed on this basis. The residuals show a marked condensation within 2′ of the centre, but from this distance outwards the representation is extremely good. According to the formula the density becomes 0·1 at the distance 27′·5 from the centre of the cluster, but this is beyond the limits of the counts on either plate. In fact it approaches the distance, 30′, at which the

density of the background is gauged. "The numbers given for the central 4 squares are estimates."

The counts (*c*) for the brighter stars of the cluster can be fairly represented by the formula beyond 2′ from the centre, the constants being $N = 18{\cdot}4$, $a = 362''$. The latter is consistent with the above value of *a*. The evidence of the series (*d*) for the fainter stars is not supported by the other counts. Otherwise it would be necessary to admit that the fainter stars do not follow the same law of distribution as the brighter. It seems curious that exactly the same allowance for the background of the sky was made both in the case of the bright and of the faint stars.

15. In all, twelve globular clusters have now been studied from the statistical point of view. Besides the present ten, ω Centauri (N.G.C. 5139) was compared with the law here taken as a standard, with results that suggested perfect conformity. Also the cluster M 5 (N.G.C. 5904) has been studied by Strömgren and Drachmann,* with results which class it as a close parallel with M 3 and therefore as belonging to the type which probably shows the greatest deviation from the normal law. Four of the ten clusters, namely, M 2, M 3, M 13, M 15, have been the subject of a most elaborate discussion by H. v. Zeipel.† Counts of the stars in 47 Toucanæ have been given previously.‡ The information relating to the five clusters M 30, M 55, M 62, N.G.C. 362 and N.G.C. 5986, is new and valuable material.

TABLE XI.

N.G.C.	*a*.	N∞.	Mag.	Centre.	Outer Region.
104	347″	5859	13–15	0–3	Very good.
362	93	1015	13–15	0	,,
5139	428	7080	13–15	...	,,
5272	113	1122	14–16	0–1	Poor.
5904	...	...	13–15	...	(As preceding.)
5986	64	481	13–16	...	Very good.
6205	146	2074	13–16	?	Fair.
6266	125	1246	14–16	0–1	Good.
6809	155	1172	12–14	...	Excellent.
7078	103	926	13–16	0–1	,,
7089	87	1140	13–16	0	Good.
7099	91	438	12–15	0–1	Excellent.

The results may fairly be summarised as in Table XI. The numbers in the third column are naturally limited to the star magnitude to which the counts extend. The magnitudes given in the fourth column have been taken from Bailey's catalogue. No precision is claimed for these, but the estimates suggest an

* *Publ. Københavns Obs.*, No. 16.
† *Kungl. Svenska Vetenskapsakad. Handl.*, B. 51, No. 5.
‡ *H.C.O. Annals*, xxvi.

astonishing degree of uniformity. The fifth column refers to the condensation of each cluster at the centre, and gives the extent over which the observed density seriously exceeds that given by the law. The central region itself is indicated by 0, and in every case concerned the observed density rests on an estimate. The distance 30″ from the centre is denoted by 1. Beyond this distance there is no marked discrepancy except in the case of 47 Toucanæ. Here it persists to a distance 90″, but, as the value of a suggests, this indicates a difference of scale, rather than a difference of structure, as compared with other clusters. In the last column of the table an attempt is made to describe in a word the degree of success with which the density in the outer parts of each cluster is represented by the law.

16. The outcome of the whole investigation is that the law of areal density (3) of § 1, and therefore the law (2) of volume density, represents the structure of a globular cluster with close approach to accuracy everywhere, with the exception of the region immediately surrounding the centre. Even this exception is not always necessary. In the clusters N.G.C. 5986, N.G.C. 6809, and probably ω Centauri, though in this case a less sensitive test (namely, by counts in successive parallel strips) was applied, the whole distribution is in harmony with this law. In other clusters an excessive density more or less marked is to be observed in the central region. It is possible, as already remarked, that this central density has been overestimated. But the deviation from the simple theory is scarcely to be explained away thus. The greatest divergence from the theory is shown by M 3: the small residuals in Table II. are only secured by a compromise which is really unsatisfactory. With M 3 must be classed the cluster M 5.

There appear to be three ways of dealing with the problem presented by the high central density which is a characteristic of some clusters:

(1) To substitute for the simple solution of the density equation (1), corresponding to the value $\gamma = 1{\cdot}2$, a similar particular solution corresponding to other values of γ. This has been done by Dr. von Zeipel in the case of the four clusters studied by him. The values of γ thus found ranged from $1{\cdot}156$ for M 3 to $1{\cdot}2$ for M 2. Even then the representation of M 3 is not very good. Professor Strömgren has prepared tables to facilitate the comparison on these lines.

(2) The particular solution hitherto admitted is defined by the central condition

$$\frac{d\rho}{dr} = 0, \qquad r = 0.$$

This condition may be abandoned and the general solution substituted for the particular solution. Dr. von Zeipel has shown that the general solution for $\gamma = 1{\cdot}2$ can be expressed in elliptic functions, and for the four clusters has found values of γ ranging between $1{\cdot}194$ and $1{\cdot}203$, so that it seems possible

to retain the precise value 1·2 at the sacrifice of the central condition.

(3) It is also possible to be content with a more modest aim and not to seek a single general law which will cover the whole distribution in every possible cluster. It is then necessary to distinguish between two classes of cluster, (*a*) open clusters, and (*b*) those possessing a distinct nuclear structure. The nucleus is generally of small extent, and outside it the distribution follows closely the standard law.

In favour of this view attention may be drawn to a remark by Professor Bailey:—"Perhaps the most striking example of this lack of symmetry is N.G.C. 6266. The total number of stars is much greater . . . in the north and preceding half of the cluster, than in . . . the south and following half. . . . This is not caused by an error in the assumed centre of the cluster, which is well defined and capable of a sufficiently precise determination. Also, this decreased density of the stars on the south following side does not appear within 60″ of the adopted centre." This seems very significant. For it suggests not only the definite character of the nucleus, but also that the centre of the nucleus is *not* coincident with the centre of the cluster as a whole. This interpretation seems in every way consistent with the facts stated. In any case, ideal representations are out of the question so long as a symmetrical law is adopted. "If density curves are derived separately for (opposite regions), the resulting curves will be found to differ from one another nearly, if not quite, as much as the mean density curves of different clusters."

Dunsink Observatory:
1915 *December* 1.

On the Motions and Distances of the Bright Stars of Type F. (Stellar Motions, No. 7.) By H. C. Plummer, M.A.

1. An earlier investigation has dealt with the bright stars of type A, for which radial velocities and proper motions are available, on the hypothesis that the peculiar motions of these stars are parallel to the galactic plane. This hypothesis, which corresponds with a general tendency first clearly stated by Kobold, was intended merely in the sense that it was sufficiently true of a sufficient number of stars to make it possible to deduce useful results by applying it. Eddington and Hartley* have recently found for type Ao the least component 5·67 directed towards a point near the galactic pole and the greatest component 14·10 in the galactic plane, and for types A1–A9 the greatest component 19·93 near the galactic plane and the least component 8·01 anywhere in a plane at right angles to the greatest component, which is an axis

* *M.N.*, lxxv. p. 521.

of symmetry. In either case the ratio of the greatest galactic component to the normal component is the same, 2·49. But there is little to show in what way this fairly high ratio is related to the distribution of the motions, whether it is due to a mixture of strongly galactic motions with others which possess marked components in the perpendicular direction, or whether it is due to a single system possessing a more or less vague tendency. The distribution of the motions in the galactic plane is not material to the hypothesis. Indeed, it should reveal itself in the result, and it will be seen later that this expectation is realised.

When we pass on to stars of more advanced type the successful application of this too rigid hypothesis apparently becomes more problematical. The prospect of useful results presented by the F type stars seemed doubtful. At this point Mr. Walkey offered some help, and at my suggestion very kindly calculated the table which is now published. The contents of this table, so far as they are the results of very simple calculation, are independent of any hypothesis. It should therefore have a certain permanent value, and may prove very useful, for example, in the search for star-streams.

2. At the same time the table renders the application of the galactic hypothesis extremely easy. It will be found that the results for the F type stars are at least as interesting as those obtained for the A type, and the investigation on these lines seems as justifiable in the one case as in the other. The hypothesis is expressed by the equation

$$\cdot 872u + \cdot 185v - \cdot 454w = 6\cdot 50,$$

where (u, v, w) are the equatorial components

$$u = \mathrm{A} + a\mathrm{D}, \quad v = \mathrm{B} + b\mathrm{D}, \quad w = \mathrm{C} + c\mathrm{D}.$$

The constant 6·50 km./sec. for the Sun's velocity normal to the galactic plane may not be the best possible, but it has been retained in conformity with the previous work. Hence

$$\frac{\varpi}{0''\cdot 01} = \frac{1}{\mathrm{D}} = \frac{\cdot 872a + \cdot 185b - \cdot 454c}{6\cdot 50 - \cdot 872\mathrm{A} - \cdot 185\mathrm{B} + \cdot 454\mathrm{C}}.$$

The values of ϖ which are not negative or obviously indeterminate are given in the third column of Table I., and the corresponding values of u, v, w in the fifth, sixth, and seventh columns. The quantities P and Q in the last two columns represent

$$\mathrm{P} = + \cdot 127u - \cdot 980v - \cdot 156w,$$
$$\mathrm{Q} = - \cdot 473u + \cdot 079v - \cdot 877w,$$

or the components towards the points $(277^\circ\cdot 4, -9^\circ)$, which is a point in the galactic plane near the commonly accepted vertex, and $(170^\circ\cdot 5, -61^\circ\cdot 3)$, which is a point in the galactic plane at right

angles to P. This is merely a convenient transformation of co-ordinates. The normal component of the peculiar velocity is

$$N = -\cdot 872u - \cdot 185v + \cdot 454w + 6\cdot 50,$$

directed towards the galactic pole (192°, +27°), and is zero by hypothesis in this case. The absolute magnitude corresponding to the standard parallax 0″·01, chosen because negative magnitudes are thereby in general avoided, is

$$m_0 = m + 5 \log (\varpi/0''\cdot 01),$$

and is recorded in the fourth column of Table I.

The number of stars available for discussion is 210. Of these 140 appear in Table I., while 70 gave negative or indeterminate parallaxes. The proportion of unfavourable cases is thus exactly one-third of the total.

Table I.

P.G.C.	Sp.	ϖ.	m_0.	u.	v.	w.	P.	Q.
12	F5	″·168	8·6	+ 8·5	+15·1	+ 8·2	−15·0	− 10·0
24	F5	·0267	7·3	+10·8	+28·2	+ 17·8	−29·0	− 18·5
120	F8	·0857	10·2	+ 1·0	+ 2·5	− 11·4	− 0·5	+ 9·7
168	F8	·464	12·0	+ 7·6	+13·1	+ 5·6	−12·7	− 7·5
321	F5	·0706	9·2	+ 3·7	+26·0	+ 3·4	−25·5	− 2·7
325	F8*p*	·0361	4·9	− 2·5	+ 5·3	− 17·0	− 2·8	+ 16·5
411	F0	·0482	8·2	+ 6·4	−14·3	− 7·8	+16·0	+ 2·7
421	F5	·0836	8·2	− 0·4	+ 0·9	− 14·8	+ 1·3	+ 13·3
543	F5	·0251	7·4	−10·6	+38·8	− 18·6	−36·4	+ 24·4
557	F2	·0078	4·9	−13·3	−75·3	− 70·7	+83·1	+ 62·4
575	F5	·0081	4·4	− 8·8	−64·9	− 57·9	+72·5	+ 49·9
626	F5	·0270	7·6	− 1·7	+67·5	+ 10·0	−67·9	+ 2·7
631	F5	·0420	7·7	− 3·7	+44·5	− 3·3	−43·6	+ 8·2
644	F0	·0192	5·7	− 9·8	+52·2	− 11·9	−50·5	+ 19·1
...*	F2	·0436	11·2	−53·4	−84·2	−151·3	+99·3	+151·3
766	G0	·814	14·7	+ 0·2	+11·9	− 9·0	−10·3	+ 8·7
772	F5	·0060	0·8	− 6·3	+25·7	− 16·0	−23·5	+ 19·0
812	F5	·140	10·5	+ 0·4	+20·7	− 5·2	−19·4	+ 6·0
847	F5*p*	·0025	0·9	+ 7·5	−18·6	− 7·4	+20·4	+ 1·5
914	F8	·0036	3·0	+12·8	−14·9	+ 4·2	+15·5	− 11·0
963	F5	·0601	8·0	+ 6·8	+14·1	+ 4·5	−13·6	− 6·0
970	Comp.	·0086	4·6	+ 2·3	+23·4	− 0·4	−22·5	+ 1·1
995	F5	·0134	5·0	+ 1·1	+73·9	+ 17·9	−75·1	− 10·4
1110	F2	·0805	9·0	+ 6·7	− 6·2	− 3·9	+ 7·6	− 0·3

* Lal. 5761.

TABLE I.—*continued.*

P.G.C.	Sp.	ϖ.	m_0.	u.	v.	w.	P.	Q.
		"						
1111	F5	·0107	5·2	+15·0	+ 76·8	+ 45·7	− 80·5	− 41·1
1140	F8	·511	11·8	+ 3·8	+ 24·9	+ 3·1	− 24·4	− 2·5
1187	F5*p*	·0028	0·2	− 6·4	+ 10·8	− 22·1	− 8·0	+ 23·3
1202	Fo	·0841	9·6	+ 2·7	+ 7·4	− 6·0	− 6·1	+ 4·6
1225	F8	·0267	6·9	+11·0	+ 12·4	+ 11·9	− 12·7	− 14·6
1290	Comp.	·0235	7·3	+ 5·1	+ 0·4	− 4·3	+ 0·9	+ 1·4
1347	F2*p*	·0026	−0·2	+ 0·8	+ 25·4	− 2·3	− 24·4	+ 3·6
1420	F8	·161	9·8	+ 6·8	− 13·3	− 6·7	+ 14·9	+ 1·6
1461	F8	·135	10·3	+ 5·9	− 12·0	− 7·8	+ 13·7	+ 3·1
1622	Fo	·0326	1·7	− 3·6	+ 13·5	− 15·7	− 11·3	+ 16·6
1725	F5	·108	8·6	+ 0·1	+ 28·7	− 2·5	− 27·7	+ 4·5
1839	F8*p*	·0130	2·5	− 7·0	+ 31·4	− 15·1	− 29·3	+ 19·0
1869	Fo	·0651	8·5	+ 9·3	+ 6·1	+ 6·1	− 5·8	− 9·2
1988	F8	·0335	7·1	−13·6	+ 57·8	− 16·9	− 55·7	+ 25·8
1999	F5	·0193	6·6	− 2·5	+ 49·9	+ 1·2	− 49·4	+ 4·0
2001	Fo	·0297	7·3	− 2·5	+ 16·8	− 12·2	− 14·9	+ 13·2
2008	F5	·871	10·2	+ 4·6	− 1·2	− 5·9	+ 2·7	+ 2·9
2036	G5	·708	14·5	− 5·1	+ 12·5	− 18·9	− 9·9	+ 20·0
2099	F8*p*	·0196	5·8	+ 1·8	+ 17·2	− 3·9	− 16·1	+ 3·9
2125	Fo	·0656	9·3	+ 9·7	− 3·7	+ 2·9	+ 4·3	− 7·4
2153	F5	·0239	4·8	− 8·3	+ 48·2	− 10·6	− 46·6	+ 17·0
2354	F8	·0332	6·1	− 3·9	+ 46·1	− 3·1	− 45·2	+ 8·1
2413	F5	·0651	8·2	0·0	+ 45·4	+ 4·2	− 45·2	− 0·1
2421	F8	·0154	5·4	+ 3·7	+ 22·3	+ 1·8	− 21·7	− 1·6
2458	F5	·174	10·7	− 4·0	+ 4·8	− 20·1	− 2·1	+ 19·9
2503	Fo	·0191	3·7	− 0·6	+ 9·1	− 11·6	− 7·2	+ 11·2
2552	F8	·150	9·1	0·0	+ 37·5	+ 0·8	− 36·8	+ 2·3
2602	F5*p*	·0212	5·4	− 3·8	+ 43·7	− 3·7	− 42·7	+ 8·5
2615	F5	·0080	4·5	− 1·1	+ 44·2	+ 1·8	− 43·7	+ 2·4
2632	F2*p*	·0249	5·9	− 2·4	+ 67·8	+ 8·8	− 68·1	− 1·2
2635	Fo	·0086	2·8	− 1·3	+ 14·1	− 11·0	− 12·3	+ 11·3
2741	F5	·0765	9·4	− 2·4	+ 18·0	− 11·5	− 16·1	+ 12·6
2768	Fo	·0087	4·5	−18·3	+ 48·7	− 29·5	− 45·4	+ 38·4
2778	F5	·0223	5·8	+ 4·9	+ 2·0	− 4·1	− 0·8	+ 1·5
2783	F5*p*	·0053	3·6	+ 6·1	− 0·3	− 2·7	+ 1·5	− 0·5
2784	Fo	·0111	4·3	+ 0·6	+ 7·2	− 10·4	− 5·4	+ 9·4
2785	F5	·0268	7·0	+ 2·7	+ 33·2	+ 4·5	− 32·9	− 2·6
2802	A5	·0156	5·8	+ 3·1	+ 45·4	+ 10·2	− 45·7	− 6·8
2813	Fo	·0277	7·4	+ 6·3	− 15·0	− 8·4	+ 16·8	+ 3·2

TABLE I.—*continued.*

P.G.C.	Sp.	ϖ.	m_0.	u.	v.	w.	P.	Q.
		"						
2830	F2	·0544	7·7	− 3·6	+ 16·7	− 14·3	− 14·7	+ 15·5
2942	Fo	·0644	8·7	+ 0·5	+ 26·3	− 2·6	− 25·3	+ 4·2
2960	F8*p*	·0065	3·1	+ 0·5	+ 14·1	− 7·6	− 12·5	+ 7·6
2966	F5*p*	·0007	−1·0	+17·6	− 71·5	− 9·7	+ 73·8	− 5·4
2999	F5	·0555	7·7	+ 5·8	− 14·5	− 9·0	+ 16·3	+ 4·1
3035	F8*p*	·0133	5·6	+13·5	− 2·8	+ 10·5	+ 2·8	− 15·8
3085	F8*p*	·0091	5·0	0·0	+ 5·4	− 12·2	− 3·4	+ 11·1
3098	F8	·0295	6·9	+ 0·7	+ 24·4	− 3·0	− 23·3	+ 4·2
3160	Fo	·0345	7·0	+ 0·4	0·0	− 13·5	+ 2·2	+ 11·6
3166	F2	·0126	4·7	+ 4·1	− 29·8	− 18·5	+ 32·6	+ 11·9
3412	F5	·0220	6·9	− 2·7	+ 96·4	+ 19·8	− 97·9	− 8·5
3427	F8	·426	13·2	+32·8	+ 13·0	+ 53·9	− 16·9	− 61·8
3558	F5	·0602	8·4	− 2·7	+ 41·0	− 2·8	− 40·1	+ 7·0
3650	Fo	·0106	5·7	−35·7	+114·6	− 35·9	−111·2	+ 57·4
3660	F5	·0673	8·3	− 8·0	− 3·6	− 31·1	+ 7·4	+ 30·8
3679	Fo	·0420	8·4	+ 7·2	− 9·8	− 4·5	+ 11·2	− 0·3
3700	F5	·0108	4·7	+ 1·6	− 0·7	− 11·6	+ 2·7	+ 9·3
3704	F8	·406	12·1	+ 0·6	+ 4·1	− 11·5	− 2·1	+ 10·1
3722	Fo	·0040	1·0	+26·5	+187·0	+112·8	−197·5	− 96·6
3729	Fo	·0531	8·1	+14·4	− 10·5	+ 9·4	+ 10·6	− 15·8
3739	Fo	·105	8·5	− 0·5	+ 11·1	− 10·9	− 9·3	+ 10·7
3758	F5	·0981	8·9	+ 0·5	− 6·2	− 15·9	+ 8·7	+ 13·2
3814	Fo	·0083	4·2	−49·1	+ 32·8	− 95·1	− 23·5	+109·2
3899	F8	·0400	8·1	+ 0·6	+ 24·2	− 3·4	− 23·1	+ 4·6
3919	Fo	·0479	8·5	− 2·3	+ 9·4	− 14·9	− 7·2	+ 14·9
3940	Fo*p*	·0435	6·9	− 0·4	+ 29·2	− 3·3	− 28·2	+ 5·4
3980	Fo	·0779	9·1	− 0·7	+ 17·9	− 8·4	− 16·3	+ 9·1
4055	F8	·212	10·5	− 2·3	− 16·1	− 25·2	+ 19·4	+ 21·9
4082	F8	·0103	4·8	− 6·8	+ 44·9	− 9·2	− 43·5	+ 14·8
4163	Fo	·0349	7·5	− 0·7	+ 50·5	+ 4·8	− 50·3	+ 0·1
4165	Fo	·0107	3·9	− 0·8	+ 47·6	+ 3·3	− 47·2	+ 1·3
4270	Fo	·0207	6·5	+ 9·4	+ 8·1	+ 7·0	− 7·8	− 9·9
4273	F5	·0864	9·4	+ 4·7	− 0·6	− 5·5	+ 2·1	+ 2·6
4320	F8	·0707	9·6	+ 0·4	+ 3·6	− 12·1	− 1·5	+ 10·7
4354	F5	·0664	9·9	− 1·4	+ 15·5	− 10·6	− 13·7	+ 11·2
4361	F2	·128	9·0	+ 7·5	+ 26·8	+ 11·0	− 27·0	− 11·0
4394	F5	·290	11·8	+ 5·6	+ 8·3	− 0·1	− 7·4	− 1·8
4420	Fo	·142	10·0	+ 5·8	+ 34·2	+ 10·7	− 34·5	− 9·4
4421	F5	·0551	8·1	− 2·8	− 25·5	− 30·0	+ 29·3	+ 25·6

TABLE I.—*continued.*

P.G.C.	Sp.	ϖ.	m_0.	u.	v.	w.	P.	Q.
		"						
4457	Fo	·0074	1·4	+ 2·8	+ 0·8	− 8·6	+ 0·9	+ 6·3
4466	F5	·146	10·7	+ 3·8	+ 1·8	− 6·4	− 0·3	+ 3·9
4492	F5*p*	·0010	−1·9	+17·8	+ 12·9	+ 25·2	− 14·2	− 29·5
4493	F8*p*	·0052	3·3	− 3·1	+ 21·7	− 11·4	− 20·9	+ 13·2
4544	Fo	·0782	9·1	+ 9·6	+ 41·7	+ 21·1	− 43·0	− 19·7
4559	Fo	·104	10·8	+ 1·4	+ 39·9	+ 4·6	− 39·6	− 1·5
4564	F5*p*	·0079	4·2	+ 5·7	+ 27·9	+ 7·9	− 27·8	− 7·4
4672	F8	·105	8·8	+25·9	− 23·0	+ 25·9	+ 21·8	− 36·8
4731	Fo	·0082	4·3	− 0·1	+ 48·4	+ 5·3	− 48·2	− 0·8
4851	F8	·0380	7·9	− 4·7	+ 63·1	+ 2·4	− 62·8	+ 5·1
4872	Fo	·0661	9·1	+ 9·1	− 0·4	+ 2·9	+ 1·1	− 6·8
4874	F2*p*	·0060	1·9	−10·0	+ 19·2	− 25·6	− 16·1	+ 28·7
4891	F5	·0158	5·9	+ 2·1	+ 33·9	+ 3·7	− 33·5	− 1·5
4929	Fo	·0022	1·0	−12·3	+ 29·1	− 26·2	− 26·0	+ 31·1
5229	F8*p*	·0020	−1·2	+ 2·7	+ 1·0	− 8·9	+ 0·7	+ 6·5
5244	Fo	·0060	3·9	− 0·6	− 18·4	− 23·0	+ 21·5	+ 19·0
5255	F5*p*	·0056	2·8	− 2·2	+ 16·0	− 12·2	− 14·1	+ 13·0
5274	F5	·0601	8·9	+ 2·3	+ 3·1	− 8·6	− 1·4	+ 6·6
5286	F8	·0333	7·9	−19·3	+ 94·1	− 12·9	− 92·7	+ 27·8
5291	F5	·0372	6·6	− 3·7	+ 26·4	− 10·7	− 24·7	+ 13·3
5328	F8	·0046	2·6	−74·9	− 1·6	−159·1	+ 16·9	+174·8
5332	Fo	·0346	7·8	+ 2·6	+ 32·3	+ 3·8	− 32·0	− 1·9
5370	Fo	·0091	5·6	+ 3·9	+ 0·8	− 6·6	+ 0·7	+ 4·0
5371	A3	·0196	6·3	+ 1·2	+ 12·2	− 7·1	− 10·7	+ 6·6
5390	F2	·0184	6·6	−52·7	+ 51·4	− 94·4	− 42·4	+111·8
5443	Fo*p*	·0448	8·0	− 5·4	+ 13·0	− 19·5	− 10·5	+ 20·7
5455	F5	·0720	8·9	− 5·9	+ 9·1	− 22·0	− 6·2	+ 22·8
5493	F8	·0554	8·0	+42·8	− 27·4	+ 56·8	+ 23·4	− 72·2
5562	Fo*p*	·0219	5·5	− 0·9	+ 51·0	+ 4·8	− 50·8	+ 0·2
5592	F5	·0114	4·5	+ 3·8	+ 13·9	− 1·4	− 12·9	+ 0·5
5623	Fo	·0482	8·6	+ 2·8	+ 34·2	+ 5·0	− 33·9	− 3·0
5688	F5	·0945	8·8	+ 3·8	+ 15·0	− 0·9	− 14·1	+ 0·2
5698	F8	·0650	9·1	+ 4·7	+ 34·5	+ 8·7	− 34·6	− 7·1
5709	F5	·0053	3·0	− 0·9	− 12·3	− 21·1	+ 15·3	+ 17·9
5874	F5	·200	10·8	− 0·2	+ 5·8	− 12·4	− 3·8	+ 11·5
5949	F5	·0523	7·9	+ 3·8	− 5·3	− 9·2	+ 7·1	+ 5·9
6024	G	·0148	5·4	− 3·9	+ 62·2	+ 3·6	− 62·1	+ 3·5
6156	F5	·0508	7·6	− 0·9	+ 14·1	− 10·3	− 12·3	+ 10·5

3. The material in Table I. relating to the values of the components P and Q may be usefully condensed into the form of a correlation diagram. We thus obtain:

TABLE II.

·1Q \ ·1P	−8	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	
+4				1									1
+3			1			1	1			1		1	5
+2				2	1	1	1	6	2		3		16
+1			1	1	4	1	5	12	11	2		1	38
0		2	1	6	3	7	7	4	7	4	4		45
−1	1			1		4	1	5	2		1		15
−2					1	1			1	1			4
−3								1					1
−4	1										1		2
	2	2	3	11	9	15	15	28	23	8	9	2	127

The limits of this table exclude 13 stars which appear in Table I. The vertical and horizontal headings indicate steps of 10 km./sec. With the material in this form an approximate calculation is greatly facilitated. We easily find:

$$n = 127, \quad \Sigma x = -210, \quad \Sigma y = +55,$$
$$\Sigma x^2 = 1038, \quad \Sigma y^2 = 235, \quad \Sigma xy = -57;$$

or, referred to the centre of gravity of equal weighted points,

$$\bar{x} = -1{\cdot}65, \quad \bar{y} = +0{\cdot}43,$$
$$\Sigma x'^2 = 693, \quad \Sigma y'^2 = 212, \quad \Sigma x'y' = +33.$$

Hence the momental ellipse is

$$212x^2 + 693y^2 - 66xy = 1,$$

of which the transverse axis is

$$x = 14{\cdot}6y.$$

This is the line of greatest extension for the distribution in the galactic plane, and it gives for the position of the vertex

$$\text{R.A. } 95^\circ{\cdot}5, \quad \text{Decl.} + 12^\circ{\cdot}5$$

if the latter point may be assumed to lie in this plane. The degree of scattering of the velocities in the direction of the vertex and in

the perpendicular direction is indicated by the axes of the momental ellipse, and we obtain

23·4 km./sec., 12·9 km./sec., ratio, 0·55.

Since those stars which are likely to have large motions normal to the galactic plane and therefore small motions in it have been excluded, the degree of scattering is probably more or less exaggerated, an effect still further increased by errors in the concluded distances of the stars. Yet these results for the galactic components are in excellent agreement with the properties of stellar motion arrived at by totally different lines of investigation.*

The motion of the system relative to the Sun is given by the centre of gravity found above, the components being

$$P = -16{\cdot}5, \quad Q = +4{\cdot}3, \quad N = 0,$$

which give for the solar motion

18·3 km./sec. towards R.A. 265°·0, Decl. + 13°·1.

The low declination of the apex for this selection of the stars of type F will be remarked.

4. The facility and apparent success of this form of discussion suggest a similar investigation for the stars of type A. The components P and Q have been given in Table IV. of Stellar Motions, No. 5 (*M.N.*, lxxiii. p. 458). With six stars excluded by the limits of the diagram, the material can be presented in the form :

TABLE III.

·1Q \ ·1P	−8	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	
+5					1										1
+4					1		1							1	3
+3				1			2	1	2		1	1			8
+2	1		1	1	2	2	6	9	1		1	1			25
+1					1	8	10	11	7	1	2		1		41
0					11	7	5	10	14	11	5	1			64
−1				1	3	2	1	4	2	2		1	1		17
−2						1		1	1						3
−3							1		1			1			3
	1	0	1	3	19	20	26	36	28	14	9	5	2	1	165

* *Cf.* Eddington and Hartley's mean result from radial velocities of 1086 stars : vertex, R.A. 94°·6, decl. + 12°·5 ; ratio of axes, 0·59.

It follows immediately from the table that

$$n = 165, \quad \Sigma x = -193, \quad \Sigma y = +100,$$
$$\Sigma x^2 = 951, \quad \Sigma y^2 = 342, \quad \Sigma xy = -162,$$

or, referred to the centre of gravity of equal weighted points,

$$\bar{x} = -1{\cdot}17, \quad \bar{y} = +0{\cdot}61,$$
$$\Sigma x'^2 = 725, \quad \Sigma y'^2 = 281, \quad \Sigma x'y' = -45.$$

Hence the momental ellipse is

$$281x^2 + 90xy + 725y^2 = 1,$$

of which the transverse axis is

$$x + 9{\cdot}97y = 0.$$

The corresponding position of the vertex, assumed to lie in the galactic plane, is

R.A. 100°·0, Decl. + 3°·7,

and the scattering of the components in the direction of the vertex and at right angles thereto in the galactic plane, as measured by the axes of the momental ellipse, is

21·0 km./sec., 12·9 km./sec., ratio, 0·61.

The resemblance between these results for the stars of type A and those already found for the stars of type F will be seen to be fairly close. They are also in excellent agreement with what we know of the galactic section of the Schwarzschild ellipsoid.

For the motion of the system relative to the Sun the centre of gravity as above determined gives the components

$$P = -11{\cdot}7, \quad Q = +6{\cdot}1, \quad N = 0,$$

whence the solar motion is found to be

14·7 km./sec. towards R.A. 264°·4, Decl. + 26°·1.

5. For the stars of type F a number of measured parallaxes are available, and a comparison of the results given in Table I. with these has a certain utility. For the whole list Mr. Walkey has supplied 33 parallaxes, but only 20 of the stars appear in Table I. The comparison is given in Table IV.

TABLE IV.

P.G.C.	Star.	ϖ_0.	ϖ_c.	ϖ_c/ϖ_0.	$5 \log \varpi_c/\varpi_0$.
		"	"		
12	β Cassiop.	·073	·168	2·30	+1·81
168	η ,,	·202	·464	2·30	+1·81
321	ω Androm.	·088	·0706	·802	−0·48
...	Lal. 5761	·038	·0436	1·15	+0·30
772	α Persei	·025	·0060	2·40	+1·90
1622	α Carinæ	·007	·0326	4·66	+3·34
1839	δ Can. Maj.	·005	·0130	2·60	+2·08
2008	α Can. Min.	·324	·871	2·69	+2·15
2413	10 Urs. Maj.	·036	·0651	1·81	+1·29
2552	θ ,,	·093	·150	1·61	+1·03
2632	υ ,,	·087	·0249	·286	−2·72
3412	42 Com. Ber.	·070	·0220	·314	−2·52
3558	τ Bootis	·042	·0602	1·43	+0·78
4055	γ Serpentis	·031	·212	6·84	+4·18
4672	χ Draconis	·114	·105	·921	−0·18
5229	γ Cygni	·054	·0020	(·037)	(−7·16)
5291	β Delphini	·010	·0372	3·72	+2·85
5455	δ Equulei	·067	·0720	1·07	+0·15
5592	κ Pegasi	·028	·0114	·407	−1·95
5874	ξ ,,	·089	·200	2·25	+1·76

Here the third column contains the absolute parallaxes deduced by Mr. Walkey from modern measures. The fourth column is taken from Table I. The ratio of the two parallaxes is shown in the fifth column, leading to the numbers in the last column, which give the effect of the discordance on the deduced absolute magnitudes.

Were the direct parallaxes a certain standard, the results of this comparison might be considered not unsatisfactory. As a matter of fact, they are not. They are affected by a considerable measure of uncertainty: in eight cases the difference from the calculated parallax is less than the probable error assigned. Moreover, the reduction from relative to absolute parallax cannot be made by any method which is not thoroughly untrustworthy. Nevertheless, with one exception (γ Cygni), the parallaxes are at least of the same order. On the whole the calculated parallaxes are the greater, making the deduced absolute magnitudes systematically fainter by nearly one magnitude. From the last column of Table IV. we find that the mean square discordance in the absolute magnitude is 2 or 2·5 according as we exclude or include γ Cygni. This may be regarded as setting an upper limit to the measure of uncertainty in the absolute magnitudes, since the errors cannot be attributed entirely to one side.

Table V.

Sp.	No.	m_0.	Light.	Sp.	No.	m_0.	Light.	Δm_0.
Fo	36	6·4	28	Fo*p*	3	6·8	19	−0·4
F2	7	7·6	9	F2*p*	3	2·5	1000	5·1
F5	44	7·6	9	F5*p*	8	1·8	1900	5·8
F8	24	8·4	4	F8*p*	8	3·6	360	4·8
F2–8	75	7·84	7	F(2–8)*p*	19	2·67	855	5·17
A5–8	19	7·6	9	Bo–2	45	2·7	832	...

6. In Table V. are shown the mean absolute magnitudes in relation to the different classes of spectrum, the material being drawn from Table I. Following the spectral class is given the number of stars involved. The absolute magnitudes (m_0) are based on the standard distance corresponding to the parallax 0″·01. There is little agreement as to the standard distance employed for this purpose by writers on the subject. I have used this standard consistently because it has the advantage of avoiding negative magnitudes in most cases. Results can readily be reduced to the parallax 0″·1 by subtracting 5 from the magnitude as given here. On the present scale the magnitude of the Sun may be taken to be 10 with sufficient approximation. Under the heading Light is given the average light of a star in terms of that radiated by the Sun as unity. In the last column is given the difference in mean magnitude between a star of normal spectral class and one of the corresponding spectral class which has been noted as "peculiar." At the bottom of the table are appended for the sake of comparison similar results previously obtained and given in Table II[1]. of Stellar Motions, No. 4 (*M.N.*, lxxiii. p. 186).

The results are remarkable. Between the classes Fo and Fo*p* there is little or no difference. This is in line with the results already obtained for the stars of types B and A, for which I have found no marked difference of magnitude depending on the spectra noted as peculiar. But for the remainder of the stars of type F the stars with peculiar spectra show a systematic difference of 5 magnitudes as compared with the normal classes. This means that a star with peculiar spectrum is on the average a hundred times brighter than a star of the same type but possessing a normal spectrum. In other words, the nature of the spectrum differentiates, at least broadly speaking, between the dwarf and giant stars with which other lines of investigation, chiefly by Hertzsprung and Russell, have already made us familiar.

The number of stars in some of the above classes is certainly small, and it might be thought that the result is the outcome of accidental error in the deduced parallaxes. But, apart from the fact that an error of 5 magnitudes implies a distance 10 times too great or too small, we have seen from the comparison with directly measured parallaxes that the mean error of a single absolute magnitude should not exceed 2. There is therefore no reason on

this ground to doubt the validity of the general conclusion that while stars with the normal F type spectrum are just about as bright as the stars of the later A type, those whose spectra have been noted as peculiar resemble in brightness the stars of the earlier B type. It may be noticed that the mean magnitude of the Fo and F*op* stars is very much the same as would result from a mixture of the later F types in the proportion found of one giant to four dwarfs, but the spectral classification has in this case failed to put this distinction in evidence.

TABLE VI.

Sp.	No.	V.	μ_0.	Sp.	No.	V.	μ_0'.	$5\log(\mu_0/\mu_0')$.
			″				″	
Fo	5	±16·5	239	F*op*	5	±20·2	100	1·78
F2	5	±25·0	171	F2*p*	3	±21·1	72	1·89
F5	10	±16·7	169	F5*p*	10	±16·8	15	5·21
F8	12	±16·6	312	F8*p*	12	±16·4	20	5·94
F2–8	27	±18·2	233	F(2–8)*p*	25	±17·1	24	4·90

7. The question is, however, of such importance that it seems desirable to test the conclusion in some way which is independent of any special hypothesis like that on which Table I. is founded. It is not proposed to enlarge the area of discussion, but to confine attention to the original list of F type stars for which the radial velocities are known. Hence in Table VI. the numbers of stars in the sixth column are those of all the stars with "peculiar" spectra occurring in the list. For each of these stars another has been taken of the corresponding normal spectrum and as near as possible in catalogue position, so as to obtain a roughly similar distribution, with the exception that the very small number in F2 has been slightly increased. Under V is given the mean radial velocity, taken without regard to sign. There is nothing in these values to suggest any marked difference of intrinsic velocity in any class. For each star the proper motion μ was found and μ_0 deduced by the relation

$$\mu_0 = \mu d, \quad \log d = 0{\cdot}2m,$$

so that, m being the apparent magnitude, μ_0 is the proper motion which the star would have were the Sun at such a distance that the magnitude of the star would be 0. The mean values are inserted in Table VI. under μ_0 and μ_0', which give centennial motions of stars of zero apparent magnitude in each class. If then there is no reason to suppose any difference of linear velocity, these motions may be taken as a relative measure of parallax, and the last column of Table VI. then represents differences of absolute magnitude between stars possessing normal and peculiar spectra, so far as they can be deduced from exceedingly limited samples.

For the classes Fo, F*op* there is a small difference of brightness which did not appear in Table V. Here on the contrary a

smaller difference is shown by the classes F2, F2*p*; this is due to the presence in the very small group F2*p* of υ Ursæ Majoris, which possesses the largest proper motion of any star represented in the right-hand side of the table. There is very fair agreement for the two later classes between the results of the two tables, and when the three classes F2–8 are combined the resulting differences of absolute magnitude agree as closely as could be expected. The result is therefore confirmed that the stars with peculiar spectra are brighter than the stars possessing the ordinary F2–8 spectrum by an amount practically equal to the whole range covered by the stars which are visible to the naked eye. It is not a mere suggestion based on an arbitrary hypothesis.*

8. It seems opportune to consider the nature of these spectral peculiarities somewhat in detail. They are based on Miss Maury's classification, and for the two chief divisions with which we are concerned, in which the evidence of Table V. and VI. agrees, we have

F5*p* = XII*c*. Representative star, ϵ Aurigæ.
F8*p* = XIII*c*. ,, ,, δ Can. Maj.

The characters in which these classes appear to coincide are—

(*a*) The hydrogen lines are narrow and well defined, and less dense than in the corresponding ordinary spectrum.

(*b*) Strong lines are prevalent and well defined. The G band is defective. But

(*c*) The behaviour of calcium is not uniform. In XII*c*. the calcium bands are narrower than in XII. In XIII*c*., on the contrary, the calcium bands attain the maximum width observed in any stellar type.

The *c* character is noted in the earlier star types, where it does not appear to correspond with any marked difference of luminosity. On the other hand, it has not been traced in the later types. Intermediate gradations between the ordinary and peculiar forms of spectrum have been observed, and it must be assumed that, in the nature of the case, the peculiar character has been assigned on somewhat rough-and-ready principles. It represents a qualitative rather than a quantitative distinction.

It is at once evident that an advance of immense possibilities becomes feasible as soon as the right key is found to open the connection between the luminosity of a star and the character of its spectrum. The *c* classification merely gives a primitive and imperfect hint. But an admirable step in the right direction has already been taken by Adams and Kohlschütter † in their work on spectral criteria for the determination of absolute stellar magnitudes. They approach the question by investigating directly

* Nor is it, as I have become aware, novel (see *Observatory*, xxxviii. p. 262). At the present time it is more than usually difficult to be cognisant of the details of recent researches.

† *Ap. J.*, xl. p. 385.

the differences of spectrum which characterise stars with small and large proper motions respectively. They find that—

(*a*) The continuous spectrum of the small proper motion stars is relatively less intense in the violet region. This effect apparently begins with the type Fo and increases as the type advances to K.

(*b*) The hydrogen lines are abnormally intense in a considerable number of small proper motion stars. This is at variance with Miss Maury's classification but may refer generally to a later range of type, where the *c* distinction breaks down.

(*c*) The relative intensities of certain pairs of lines are independent of proper motion, and furnish a quantitative subdivision of the Draper classification. The relative intensities of certain other pairs do depend on proper motion (or parallax), and furnish the means for the determination of absolute magnitude. The more important part of the paper deals with the application of this test.

Exactly how far these criteria are trustworthy and how far or in what way they are capable of extension to earlier and later types can be tested only by ampler experience. But it is now clear that a stellar spectrum requires at least two distinct indices for its adequate specification. When these are available the prospect of deriving absolute magnitudes and approximate parallaxes has become distinctly promising. But when the secondary index is unknown mean parallaxes and parallax formulæ are apt to be not merely useless but actually misleading.

9. A few of the stars may be attributed to that group associated with Ursa Major, the motions of which are characteristic of Stream II. They are given in Table VII.

TABLE VII.

P.G.C.	Star.	ϖ.	u.	v.	w.
		"			
411	χ Ceti	·0482	+6·4	−14·3	−7·8
1420	γ Leporis	·161	+6·8	−13·3	−6·7
1461	χ_1 Orionis	·135	+5·9	−12·0	−7·8
2813	37 Urs. Maj.	·0277	+6·3	−15·0	−8·4
2999	ι Leonis	·0555	+5·8	−14·5	−9·0
...	Gr. 1930	(·025)	+9·5	−12·7	−7·6
3382	78 Urs. Maj.	(·030)	+4·5	−14·9	−7·9

The first five of these stars are taken directly from Table I. without any change. Their motions as given satisfy the galactic hypothesis. But others differing little from them might not satisfy the hypothesis. A search was therefore made through Mr. Walkey's table to find such as might fairly be brought in. This illustrates the value of that table, for the process is extremely simple. The result yielded two stars only, which are placed at the end of Table VII. with the parallaxes required to bring their motions into line. It is sometimes supposed that among a considerable number of stars with undetermined parallaxes it must be easy to find several

which can be made to conform with a prescribed type of motion. Experience of this kind suggests the contrary.

Of the above seven stars three were suggested by Hertzsprung * as probable members of this extended group. They are: 37 Ursæ Maj. ($\varpi = ''\cdot045$), Gr. 1930 ($\varpi = ''\cdot028$), and 78 Ursæ Maj. ($\varpi = ''\cdot042$). The other four, χ Ceti, γ Leporis, χ_1 Orionis, and ι Leonis are now suggested for the first time. There is every reason why γ Leporis and χ_1 Orionis should find a place on the observing list for the measurement of their parallaxes.†

These stars have been classed together on account of the close resemblance and comparatively isolated type of their motions. They have also been identified with the Ursa Major group because they include the three F type stars suggested by Hertzsprung. But they give the velocity, relative to the Sun,

17·2 km./sec. towards R.A. 295°·0, Decl. – 27°·4,

as compared with

18·4 km./sec. towards R.A. 307°·8, Decl. – 40°·2 (Hertzsprung)

and

20·7 km./sec. towards R.A. 303°·2, Decl. – 36°·6 (Ludendorff).‡

The agreement is but moderate. Yet the difference may have no significance. As a matter of fact, the radial velocity of 78 Ursæ Maj. was – 13·0 according to Hertzsprung's computation, but its observed value is – 7·0. In his position of the convergent the proper motion of Sirius is given overwhelming weight. Doubtless its direction is very well determined. But unless the motions of the group are rigidly parallel, which is unlikely, this preferential treatment of one proper motion seems unjustified.

* *Ap. J.*, xxx. p. 135.

† Another parallax which needs determination is that of θ Cassiop. (A5). The only determination which has been made is $>0''\cdot2$, which is most unlikely but should be confirmed or disproved.

‡ *A.N.* 4313-4.

Dunsink Observatory,
1915 *December* 6.

Table of Stellar Motions in Equatorial Co-ordinates and other Data for Stars of Type F. By the Rev. O. R. Walkey.

In accordance with the scheme proposed by Professor Plummer on lines somewhat similar to his papers on the motions of the B and A type stars, I have treated the stars of type F as herein detailed.

The contents of the following table may be described according to the several columns thus :—

Column (1).—The number of the star in the Preliminary General Catalogue of Lewis Boss.

Column (2).—The corresponding number in the Revised Harvard Photometry (*H.C.O. Annals*, 1).

Column (3).—Name of the star. When the star has no name in the accepted sense, the constellation is generally given as a rough indication of the region of the sky to which the star belongs.

Column (4).—Spectrum of the star, expressed in tenths from Fo towards Go. Thus F – 7 means A3, F15 means G5.

The occurrence of such anomalous designations in a list purporting to refer to F type stars must be explained. All the stars for which the radial velocities are given in *L.O.B.*, 229, and are there attributed to type F, appear in the present list. But their spectra have been revised after comparison with *H.C.O. Annals*, l. and lvi. At the same time, as a result of a similar revision of their spectra, eleven additional stars now attributed to type F have been transferred from the lists of radial velocities given in *L.O.B.*, 195 and 211.

Column (5).—The visual magnitude of the star, taken from the R.H.P.

Column (6).—Approximate R.A. (1900·0) in arc, derived from the P.G.C.

Column (7).—Approximate declination (1900·0), contracted from the P.G.C.

Column (8).—$\mu_1 = 15\mu_\alpha \cos\delta$, the annual proper motion in R.A., expressed in seconds of arc and derived from the P.G.C.

Column (9).—μ_2, or the annual proper motion in declination, also in seconds of arc, taken from the P.G.C.

Column (10).—V, the radial velocity of the star in km./sec., taken from the lists published in the *L.O.B.*, as explained in the above note to the description of column (4).

Column (11).—$A = V \cos\delta \cos\alpha$.

Column (12).—$a = -473{\cdot}7\ (\mu_1 \sin\alpha + \mu_2 \sin\delta \cos\alpha)$.

Column (13).—$B = V \cos\delta \sin\alpha$.

Column (14).—$b = 473{\cdot}7\ (\mu_1 \cos\alpha - \mu_2 \sin\delta \sin\alpha)$.

Column (15).—$C = V \sin\delta$.

Column (16).—$c = 473{\cdot}7\ \mu_2 \cos\delta$.

The main purpose of the table, beyond collecting the relevant data in the most convenient form, is to give the most direct means of finding the components of linear velocity in km./sec., referred

to the equatorial system of rectangular axes. These axes are drawn (1) through the point (0^h, $0°$); (2) through the point (6^h, $0°$); and (3) through the N. Pole. Then if (u, v, w) are the corresponding components of a star's velocity relative to the Sun, and D is its distance expressed in the unit 100 parsecs,

$$u = A + aD, \quad v = B + bD, \quad w = C + cD$$

where

$$D = 0''{\cdot}01/\varpi$$

and ϖ is the parallax of the star. Thus the components can be derived immediately from the table when the parallax is known.

The above forms will be easily understood. Let M_1, M_2 be the linear velocities corresponding to μ_1, μ_2, expressed in the same unit as V. Then the relations between the rectangular set (V, M_1, M_2) and the set (u, v, w) are clearly given by the scheme:

	V.	M_1.	M_2.
u	$\cos\delta\cos\alpha$	$-\sin\alpha$	$-\sin\delta\cos\alpha$
v	$\cos\delta\sin\alpha$	$\cos\alpha$	$-\sin\delta\sin\alpha$
w	$\sin\delta$	0	$\cos\delta$

But

$$M_1 = 4{\cdot}737\mu_1/\varpi, \quad M_2 = 4{\cdot}737\mu_2/\varpi,$$

ϖ being expressed in seconds of arc, since an annual proper motion of $1''$ is the equivalent of 4·737 km./sec. when the parallax is $1''$.

The only other explanation necessary is that for the three stars not given in the P.G.C., the proper motion of SU Cygni is insensible, while for Lal. 5761 and R.H.P. 4867 the p.ms. have been taken from Greenwich and Cincinnati Catalogues.

I must express my deep indebtedness to Professor Plummer for his trouble and kindness in preparing and checking my MS. for publication.

(1) P.G.C.	(2) R.H.P.	(3) Star.	(4) Sp. F.	(5) Mag	(6) R.A.	(7) Decl.	(8) μ_1.	(9) μ_2.	(10) V.	(11) A.	(12) a.	(13) B.	(14) b.	(15) C.	(16) c.
12	21	β Cassiop.	5	2·42	0 58	+58 36	+·528	−·181	+12·8	+ 6·7	+ 31	+ 0·1	+252	+10·9	− 45
24	35	θ Sculpt.	5	5·19	1 40	−35 42	+·157	+·120	− 1·0	− 0·8	+ 31	0·0	+ 75	+ 0·6	+ 46
55	77	ζ Toucan.	8	4·34	3 43	−65 28	+1·696	+1·166	+ 9·3	+ 3·9	+450	+ 0·3	+834	− 8·5	+229
116	142	13 Ceti	0	5·24	7 32	− 4 9	+·407	−·018	+10·5	+10·4	− 26	+ 1·4	+191	− 0·8	− 9
120	147	λ_2 Phœn.	8	5·52	7 44	−48 33	+·042	−·118	+ 9·4	+ 6·2	− 44	+ 0·8	+ 14	− 7·0	− 37
143	183	ξ ,,	op	5·83	9 18	−57 3	+·082	+·038	+11·3	+ 6·1	+ 9	+ 1·0	+ 41	− 9·5	+ 10
168	219	η Cassiop.	8	3·64	10 46	+57 17	+1·127	−·522	+10·0	+ 5·3	+105	+ 1·0	+564	+ 8·4	−134
187	242	ρ Phœn.	5	5·22	11 32	−51 32	+·051	+·038	+26	+15·8	+ 9	+ 3·2	+ 27	−20·4	+ 11
189	244	Bradley 90	8	4·93	11 46	+60 35	−·072	+·174	+21·3	+10·2	− 63	+ 2·1	− 48	+18·6	+ 41
294	377	κ Toucan.	8	4·96	18 6	−69 24	+·396	+·116	+12	+ 4·0	− 9	+ 1·3	+195	−11·2	+ 19
321	417	ω Androm.	5	4·96	20 25	+44 53	+·344	−·096	+11·3	+ 7·5	− 27	+ 2·8	+164	+ 8·0	− 32
325	424	α Urs. Min.	$8p$	2·12	20 38	+88 46	+·042	+·001	−17	− 0·3	− 8	− 0·1	+ 20	−17·0	0
372	483	— Androm.	0	5·10	23 55	+42 7	+·807	−·142	+ 4·9	+ 3·3	−114	+ 1·5	+368	+ 3·3	− 50
411	531	χ Ceti	0	4·77	26 10	−11 11	−·159	−·079	+ 1·0	+ 0·9	+ 27	+ 0·4	− 71	− 0·2	− 37
421	544	α Triang.	5	3·58	26 51	+29 6	+·017	−·232	−7	− 5·5	+ 44	− 2·8	+ 31	− 3·4	− 96
458	591	α Hydri	0	3·02	28 54	−62 3	+·250	+·044	−5	− 2·1	− 41	− 1·1	+112	+ 4·4	+ 10
543	695	κ Fornac.	5	5·37	34 30	−24 16	+·201	−·061	+19·8	+14·9	− 64	+10·2	+ 72	− 8·1	− 26
557	714	λ Horolog.	2	5·47	35 32	−60 46	−·072	−·135	+35	+13·9	− 21	+ 9·9	− 66	−30·5	− 31
575	740	σ Ceti	5	4·82	36 50	−15 41	−·064	−·117	−29	−22·3	+ 11	−16·7	− 39	+ 7·8	− 53
605	781	ϵ ,,	5	5·01	38 41	−12 18	+·151	−·236	+16	+12·2	− 63	+ 9·8	+ 41	− 3·4	−109

618	802	ζ Horolog.	2	5·26	39 23	−54 59	+ ·034	− ·006	−1	− 0·4	− 12	− 0·4	+ 11	+ 0·8	− 2
626	810	ι ,,	5	5·42	39 47	−51 14	+ ·301	+ ·222	+18·4	+ 8·9	− 28	+ 7·4	+163	−14·3	+ 66
631	818	τ_1 Erid.	5	4·61	40 7	−19 0	+ ·323	+ ·047	+25·5	+18·4	− 93	+15·5	+122	− 8 3	+ 21
644	840	16 Persei	0	4·27	41 4	+37 54	+ ·186	− ·105	+14	+ 8·3	− 35	+ 7·3	+ 86	+ 8·6	− 39
706	934	μ Horolog.	0	5·16	45 19	−60 8	− ·074	− ·066	+20	+ 7·0	+ 6	+ 7·1	− 44	−17·3	− 16
..	...	Lal. 5761	2	8·0	45 38	+25 58	− ·191	− ·864	−153	−96·2	+190	−98·2	+ 65	−67·0	−368
723	963	α Fornac.	8	3·95	46 57	−29 23	+ ·327	+ ·648	−20·6	−12·3	− 10	−13·1	+216	+10·1	+268
766	1010	ζ_2 Retic.	10	5·16	49 1	−62 53	+1·315	+ ·692	+12·2	+ 3·6	−279	+ 4·2	+629	−10·9	+149
772	1017	α Persei	5	1·90	49 18	+49 30	+ ·027	− ·028	− 2·2	− 0·9	− 3	− 1·1	+ 16	− 1·7	− 9
812	1083	κ Retic.	5	4·80	51 54	−63 17	+ ·365	+ ·369	+12·1	+ 3·4	− 40	+ 4·3	+230	−10·8	+ 79
847	1135	ν Persei	5*p*	3·93	54 36	+42 16	− ·009	+ ·001	−12·9	− 5·5	+ 3	− 7·8	− 3	− 8·7	0
873	1173	τ_6 Erid.	8	4·33	55 38	−23 33	− ·162	− ·523	+ 7·3	+ 3·8	+ 7	+ 5·5	−125	− 2·9	−227
914	1230	— Cephei	8	5·25	58 19	+80 25	− ·014	+ ·005	+ 3·2	+ 0·3	+ 5	+ 0·5	− 6	+ 3·2	0
955	1287	ρ Tauri	0	5·55	61 11	+26 13	− ·027	− ·037	+19·6	+ 8·5	+ 15	+15·4	+ 1	+ 8·7	− 16
963	1298	o_1 Erid.	5	4·14	61 45	− 7 6	+ ·009	+ ·081	+15	+ 7·0	− 2	+13·1	+ 6	− 1·9	+ 38
970	1306	*f* Persei	Comp.	4·89	62 1	+40 14	+ ·016	− ·026	+16·4	+ 5·9	− 3	+11·1	+ 11	+10·6	− 9
995	1338	γ Dorad.	5	4·36	63 21	−51 44	+ ·091	+ ·180	+27	+ 7·5	− 8	+14·9	+ 79	−21·2	+ 53
1110	1502	α Cæli	2	4·52	69 20	−42 3	− ·145	− ·089	0	0	+ 54	0	− 51	0	− 31
1111	1503	β ,,	5	5·08	69 38	−37 20	+ ·029	+ ·187	+33	+ 9·1	+ 6	+24·6	+ 55	−20·0	+ 70
1140	1543	π_3 Orion.	8	3·31	71 6	+ 6 47	+ ·471	+ ·020	+25	+ 8·0	−211	+23·5	+ 71	+ 3·0	+ 9

(1) P.G.C.	(2) R.H.P.	(3) Star.	(4) Sp. F.	(5) Mag.	(6) R.A.	(7) Decl.	(8) μ_1.	(9) μ_2.	(10) V.	(11) A.	(12) a.	(13) B.	(14) b.	(15) C.	(16) c.
					h m	° ′	″	″							
1187	1605	ϵ Aurig.	5*p*	3+var.	73 42	+43 41	+·005	−·013	−9	−1·8	−1	−6·3	+5	−6·2	−4
1202	1637	9 ,,	o	4·99	74 43	+51 28	−·015	−·177	+0·2	0·0	+24	+0·1	+61	+0·2	−52
1207	1649	η_1 Pictor	5	5·44	75 3	−49 18	−·055	+·013	+22·1	+3·7	+26	+13·9	−2	−16·8	+4
1225	1674	ζ Dorad.	8	4·76	75 57	−57 37	−·043	+·105	−2·3	−0·3	+30	−1·1	+36	+1·9	+27
1227	1676	15 Orion.	o	4·86	76 0	+15 28	+·004	−·010	+32·7	+7·6	−2	+30·6	+2	+8·7	−5
1235	1686	— Camel.	8	5·16	76 31	+79 7	−·080	+·155	−9·7	−0·4	+20	−1·8	−79	−9·5	+14
1290	1771	— Lepor.	10	5·14	79 25	−24 52	−·024	−·017	+3	+0·5	+11	+2·7	−5	−1·3	−7
1347	1865	α ,,	2*p*	2·69	82 5	−17 54	+·001	+·003	+24·9	+3·2	−1	+23·4	+1	−7·7	+1
1384	1922	β Dorad.	5*p*	3·81	83 11	−62 33	−·007	+·014	+12	+0·7	+4	+5·5	+5	−10·6	+3
1420	1983	γ Lepor.	8	3·80	85 4	−22 29	−·278	−·376	−9·1	−0·7	+126	−8·4	−79	+3·5	−165
1461	2047	χ_1 Orion.	8	4·62	87 7	+20 15	−·186	−·094	−13·7	−0·6	+89	−12·8	+11	−4·7	−42
1476	2085	η Lepor.	5	3·77	87 58	−14 11	−·041	+·132	+0·3	0·0	+20	+0·3	+15	−0·1	+61
1622	2326	α Carin.	o	−0·86	95 26	−52 38	+·015	+·010	+20·8	−1·2	−8	+12·6	+3	−16·5	+3
1671	2400	— ,,	8	5·60	97 14	−51 45	+·078	+·092	+16·7	−1·3	−37	+10·3	−1	−13·1	+27
1725	2484	ξ Gem.	5	3·40	99 55	+13 0	−·114	−·201	+27	−4·5	+50	+25·9	+30	+6·1	−93
1789	2590	π Can. Maj.	5	4·62	102 49	−20 1	+·056	+·036	+6	−1·3	−27	+5·5	0	−2·1	+16
1839	2693	δ ,,	8*p*	1·98	106 5	−26 14	−·005	+·002	+35·5	−8·8	+2	+30·6	+1	−15·7	+1
1869	2740	I Puppis	o	4·47	107 26	−46 36	−·148	+·089	−2·3	+0·5	+57	−1·5	+50	+1·7	+29
1917	2803	δ Volant.	5*p*	4·02	109 13	−67 46	−·020	−·006	+22·7	−2·8	+10	+8·1	+1	−21·0	−1
1952	2852	ρ Gem.	o	4·18	110 40	+31 59	+·149	+·183	−4	+1·2	−50	−3·2	−68	−2·1	+74

1988	2906	— Puppis	8	4·52	112 27	−22 5	− ·067	+ ·045	+60·8	−21·4	+ 26	+52·1	+ 19	−22·9	+ 20
1999	2927	25 Monoc.	5	5·17	113 5	− 3 53	− ·070	+ ·018	+46·7	−18·3	+ 30	+42·9	+ 14	− 3·2	+ 9
2001	2930	o Gem.	o	4·92	113 10	+34 49	− ·027	− ·122	+ 6·5	− 2·1	− 1	+ 4·9	+ 35	+ 3·7	− 47
2008	2943	α Can. Min.	5	0·48	113 31	+ 5 29	− ·696	−1·030	− 3·5	+ 1·4	+284	− 3·2	+174	− 0·3	−485
2036	2998	— Puppis	15	5·22	114 58	−44 55	− ·072	− ·568	+23	− 6·9	+111	+14·8	−158	−16·2	−191
2099	3102	*j* ,,	8*p*	4·35	118 8	−22 37	− ·040	+ ·008	+14·7	− 6·4	+ 16	+12·0	+ 10	− 5·7	+ 4
2125	3140	— ,,	o	5·20	118 59	−39 1	− ·092	− ·038	− 8·0	+ 3·0	+ 44	− 5·4	+ 11	+ 5·0	− 14
2153	3185	ρ ,,	5	2·88	120 49	− 24 1	− ·089	+ ·045	+46	−21·5	+ 32	+36·1	+ 29	−18·7	+ 19
2168	3208	ζ_1 Canc.	10	5·56	121 37	+17 57	+ ·066	− ·140	− 5·1	+ 2·5	− 37	− 4·1	+ 1	− 1·6	− 63
2177	3220	B Carin.	5	4·80	121 50	− 61 0	− ·152	− ·288	+25·5	− 6·5	+124	+10·5	− 63	−22·3	− 66
2244	3318	α Cham.	5	4·08	125 17	−76 36	+ ·090	+ ·115	−13·9	+ 1·9	− 65	− 2·6	+ 19	+13·5	+ 13
2354	3482	ε Hydræ	8	3·48	130 22	+ 6 47	·189	− ·054	+37	−23·8	+ 66	+28·0	+ 60	+ 4·4	− 25
2413	3579	10 Urs. Maj.	5	4·09	133 32	+42 11	− ·431	− ·261	+27·3	−13·9	+ 91	+14·7	+201	+18·3	− 92
2421	3591	W Velor.	8	4·42	134 5	− 40 52	− ·059	+ ·033	+9	− 4·7	+ 13	+ 4·9	+ 27	− 5·9	+ 12
2427	3598	b_2 Carin.	o	5·17	134 14	−58 42	− ·184	+ ·270	+7	− 2·5	− 14	+ 2·6	+139	− 6·0	+ 66
2441	3616	σ_2 Urs. Maj.	8	4·87	135 24	+67 32	− ·005	− ·070	− 1·8	+ 0·5	− 20	− 0·5	+ 23	− 1·7	− 13
2446	3624	τ ,,	8*p*	4·74	135 40	+63 55	+ ·102	− ·069	−6	+ 1·9	− 55	− 1·8	− 14	− 5·4	− 14
2458	3643	G Carin.	5	4·50	136 13	−72 12	− ·025	− ·008	+21·0	− 4·6	+ 11	+ 4·4	+ 6	−20·0	− 1
2503	3699	ι ,,	o	2·25	138 36	−58 51	− ·026	− ·002	+13·3	− 5·2	+ 9	+ 4·5	+ 9	−11·4	o
2540	3757	*h* Urs. Maj.	o	3·75	140 55	+63 30	+ ·114	+ ·025	−11	+ 3·8	− 26	− 3·1	− 50	− 9·8	+ 5

(1) P.G.C.	(2) R.H.P.	(3) Star.	(4) Sp. F.	(5) Mag.	(6) R.A.	(7) Decl.	(8) μ_1.	(9) μ_2.	(10) V.	(11) A.	(12) a.	(13) B.	(14) b.	(15) C.	(16) c.
2541	3759	τ_1 Hydræ	5	4·78	141° 1′	− 2° 20′	+ ″·126	− ″·019	+11·1	− 8·6	− 37	+ 7·0	− 47	− 0·5	− 9
2552	3775	θ Urs. Maj.	8	3·26	141 33	+52 8	− ·945	− ·549	+15·7	− 7·5	+117	+ 6·0	+478	+12·4	−160
2602	3852	o Leonis	5p	3·76	143 57	+10 21	− ·145	− ·039	+27·1	−21·5	+ 38	+15·7	+ 57	+ 4·9	− 18
2615	3871	θ Antl.	5	4·98	144 56	−27 19	− ·060	+ ·023	+22·6	−16·4	+ 12	+11·5	+ 26	−10·4	+ 10
2632	3888	υ Urs. Maj.	2p	3·89	145 58	+59 31	− ·289	− ·159	+28	−11·8	+ 23	+ 8·0	+150	+24·1	− 38
2635	3890	υ Carin.	o	3·15	146 9	−64 36	− ·020	+ ·006	+13·8	− 4·9	+ 3	+ 3·3	+ 9	−12·5	+ 1
2741	4054	40 Leonis	5	4·97	153 34	+19 59	− ·234	− ·231	+ 5·6	− 4·7	+ 16	+ 2·3	+116	+ 1·9	−103
2768	4090	30 Leo. Min.	o	4·83	155 3	+34 18	− ·067	− ·083	+14	−10·5	− 7	+ 4·9	+ 38	+ 7·9	− 33
2778	4102	I Carin.	5	4·08	155 36	−73 31	− ·018	− ·021	+ 3	− 0·8	+ 12	+ 0·4	+ 4	− 2·9	− 3
2783	4110	— ,,	5p	4·94	155 55	−57 8	− ·002	− ·007	− 0·8	+ 0·4	+ 3	− 0·2	0	+ 0·7	− 2
2784	4114	S ,,	o	4·08	156 3	−58 14	− ·016	− ·008	+10·1	− 4·9	+ 6	+ 2·2	+ 6	− 8·6	− 2
2785	4112	36 Urs. Maj.	5	4·84	156 3	+56 30	− ·178	− ·038	+ 9·8	− 4·9	+ 21	+ 2·2	+ 83	+ 8·2	− 10
2802	4132	— ,,	−5	4·84	156 51	+40 56	− ·141	− ·007	+18	−12·5	+ 24	+ 5·3	+ 63	+11·8	− 3
2813	4141	37 ,,	o	5·16	157 11	+57 36	+ ·067	+ ·031	−13·3	+ 6·6	− 1	− 2·8	− 34	−11·2	+ 8
2830	4167	p Velor.	2	4·06	158 17	−47 42	− ·164	− ·030	+17	−10·6	+ 38	+ 4·2	+ 68	−12·6	− 10
2942	4310	χ Leonis	o	4·66	164 58	+ 7 53	− ·346	− ·047	+ 6·0	− 5·7	+ 40	+ 1·5	+159	+ 0·8	− 22
2960	4337	x Carin.	8p	4·02	166 5	−58 26	− ·019	− ·002	+ 8	− 4·1	+ 3	+ 1·0	+ 8	− 6·8	− 1
2966	4352	y ,,	5p	4·73	167 5	−59 46	+ ·098	− ·005	− 8·3	+ 4·1	+ 1	− 0·9	− 5	+ 7·2	− 1
2999	4399	ι Leonis	5	4·03	169 41	+11 5	+ ·155	− ·085	− 9·9	+ 9·6	− 21	− 1·7	− 71	− 1·9	− 40
3035	4441	o_1 Cent.	8p	4·96	171 47	−58 53	+ ·002	− ·018	−16	+ 8·2	+ 7	− 1·2	− 2	+13·7	− 4

3085	4511	— ,,	8*p*	5·18	174	41	−61	56	− ·011	− ·010	+11	− 5·1	+ 5	+ 0·5	+ 5	− 9·7	− 2
3098	4527	93 Leonis	8	4·54	175	42	+20	46	− ·154	− ·020	0·0	0·0	+ 2	0·0	+ 73	0·0	− 9
3105	4540	*β* Virg.	8	3·80	176	22	+ 2	20	+ ·742	− ·279	+ 4·9	− 4·9	− 28	+ 0·3	−351	+ 0·2	−132
3160	4616	*η* Crucis	0	4·30	180	25	−64	3	·000	− ·046	+12·0	− 5·3	+ 20	0·0	0	−10·8	− 10
3166	4623	*α* Corvi	2	4·18	180	49	−24	10	+ ·079	− ·049	+ 4·2	− 3·8	+ 10	− 0·1	− 37	− 1·7	− 21
3307	4825-6	*γ* Virg.	0 0	3·65 3·68	189	9	− 0	54	− ·564	+ ·004	−20·0	+19·7	− 42	+ 3·2	+264	+ 0·3	+ 2
...	4867	— Urs. Maj.	0	5·87	191	5	+60	52	+ ·075	+ ·012	− 10	+ 4·8	+ 12	+ 0·9	− 34	− 8·7	+ 3
3382	4931	78 ,,	0	4·89	194	7	+56	54	+ ·098	− ·023	− 7·0	+ 3·7	+ 2	+ 0·9	− 47	− 5·9	− 6
3412	4968-9	42 Com. Ber.	5	5·22 5·22	196	17	+18	3	− ·432	+ ·122	−16·8	+15·3	− 40	+ 4·5	+202	− 5·2	+ 55
3427	4989	— Cent.	8	5·04	197	1	−58	34	− ·256	− ·175	−64·4	+32·1	+ 32	+ 9·8	+137	+54·9	− 43
3447	5017	20 Can. Ven.	0	4·66	198	16	+41	6	− ·125	+ ·004	+ 9·5	− 6·8	− 17	− 2·2	+ 57	+ 6·2	+ 1
3518	5127	25 ,,	0	4·92	203	15	+36	48	− ·103	+ ·014	− 6	+ 4·4	− 16	+ 1·9	+ 47	− 3·6	+ 5
3544	5168	*i* Cent.	5	4·36	205	0	−32	32	− ·465	− ·153	−14·6	+11·2	− 58	+ 5·4	+216	+ 7·9	− 61
3558	5185	*τ* Bootis	5	4·51	205	38	+17	57	− ·485	+ ·026	−15·4	+13·2	− 96	+ 6·3	+209	− 4·8	+ 12
3635	5304	*d* ,,	5	4·82	211	28	+25	34	− ·023	− ·072	+ 9·8	− 7·5	− 18	− 4·6	+ 2	+ 4·2	− 31
3650	5323	14 ,,	0	5 54	212	19	+13	26	− ·260	− ·062	−38·6	+31·7	− 72	+20·1	+100	− 9·0	− 29
3660	5338	*ι* Virg.	5	4 16	212	42	− 5	31	− ·018	− ·427	+11·6	− 9·7	+ 12	− 6·2	+ 18	− 1·1	−201
3679	5365	18 Bootis	0	5·31	213	36	+13	28	+ ·108	− ·038	− 1·6	+ 1·3	+ 25	+ 0·9	− 45	− 0·4	− 18
3700	5396	τ_2 Lupi	5	4·49	214	56	−44	56	+ ·003	− ·020	+ 7·6	− 4·4	+ 6	− 3·1	+ 3	− 5·4	− 7
3704	5404	*θ* Bootis	8	4·06	215	27	+52	19	− 238	− ·406	−10·9	+ 5·4	−190	+ 3·9	+ 4	− 8·6	−118

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3722	5435	γ Bootis	o	3·00	217° 1′	+38° 45′	−″·111	+″·144	−35	+21·8	+2	+16·4	+68	−21·9	+53
3729	5447	σ ,,	o	4·48	217 35	+30 11	+·193	+·120	+0·3	−0·2	+78	−0·2	−55	+0·2	+49
3739	5463	α Circ.	o	3·41	218 36	64 32	−·200	−·236	+7·0	−2·4	+20	−1·9	+137	−6·3	−48
3758	5487	μ Virg.	5	3·95	219 27	−5 13	+·106	−·322	+5	−3·8	+43	−3·2	−30	−0·5	−152
3814	5570	16 Libræ	o	4·59	222 59	−3 56	−·100	−·164	+20	−14·6	−29	−13·6	+38	−1·4	−77
3855	5634	*c* Bootis	o	5·03	225 44	+25 16	+·187	−·183	−7	+4·4	+38	+4·5	−88	−3·0	−78
3871	5660	*i* Lupi	o	4·95	227 7	−31 9	+·004	−·018	−22·5	+13·1	+4	+14·1	+2	+11·6	−7
3878	5667	— ,,	Comp.	5·20	227 22	−41 7	−·005	−·017	−26	+13·3	+2	+14·4	+5	+17·1	−6
3899	5698	ν_1 ,,	8	5·06	228 48	−47 34	−·138	−·141	−10·7	+4·8	−17	+5·4	+80	+7·9	−45
3919	5723	ϵ Libræ	o	5·08	229 42	−9 58	−·080	−·159	−3	+1·9	−20	+2·3	+34	+0·5	−74
3926	5733	μ Bootis	o	4·47	230 11	+37 44	−·149	+·078	−8	+4·1	−40	+4·9	+63	−4·9	+29
3940	5747	β Cor. Bor.	o*p*	3·72	230 56	+29 27	−·174	+·076	−21·3	+11·7	−53	+14·4	+66	−10·5	+31
3980	5825	*g* Lupi	o	4·69	233 35	−44 20	−·184	−·266	−4·6	+2·0	−18	+2·6	+123	+3·2	−90
4042	5914	χ_1 Herc.	10	4·61	237 18	+42 44	+·442	+·619	−55·6	+22·1	+284	+34·3	+55	−37·8	+215
4055	5933	γ Serp.	8	3·86	237 57	+15 59	+·303	−1·297	+7·3	−3·7	+32	−5·9	−220	+2·0	−591
4082	5978	ξ Scorpii	8	4·77	239 43	−11 6	−·062	−·034	−32·9	+16·3	−24	+27·9	+17	+6·3	−16
4090	5986	θ Dracon.	8	4·11	240 0	+58 50	−·311	+·338	−8·4	+2·2	−59	+3·8	+193	−7·2	+83
4135	6058	γ_1 Norm.	8*p*	5·00	242 23	−49 49	−·006	−·004	−17·2	+5·1	−2	+9·8	+3	+13·1	−1
4163	6093	σ Serp.	o	4·80	244 15	+1 16	−·166	+·043	−45	+19·5	−71	+40·5	+35	−1·0	+20
4165	6095	γ Herc.	o	3·79	244 23	+19 23	−·048	+·039	−39	+15·9	−18	+33·2	+15	−12·9	+17

4206	6153	ω Oph.	o	4·57	246	33	−21	15	+ ·021	+ ·026	+ 2·6	− 1·0	+ 7	− 2·2	− 8	− 0·9	+ 12
4270	6237	— Dracon.	o	4·88	250	51	+56	58	+ ·027	+ ·056	o	0·0	+ 19	0·0	+ 17	0·0	+ 15
4273	6243	20 Oph.	5	4·73	251	5	−10	36	+ ·086	− ·102	+ 0·1	0·0	+ 41	− 0·1	− 5	0·0	− 47
4320	6314	ϵ_2 Aræ	8	5·36	253	47	−53	5	− ·009	− ·151	+ 7·5	− 1·3	+ 12	− 4·3	+ 56	− 6·0	− 43
4354	6369–70	μ Dracon.	5	$\left\{\begin{matrix}5\cdot83\\5\cdot80\end{matrix}\right\}$	255	49	+54	36	− ·072	+ ·081	−17·2	+ 2·4	− 25	+ 9·7	+ 39	−14·0	+ 22
4361	6380	η Scorpii	2	3·44	256	15	−43	6	+ ·024	− ·294	−28	+ 4·9	+ 34	+19·9	+ 90	+19·1	−102
4394	6445	ξ Oph.	5	4·46	258	45	−21	0	+ ·239	− ·207	− 8·6	+ 1·6	+118	+ 7·9	+ 12	+ 3·1	− 91
4420	6486	b ,,	o	4·28	260	4	−24	5	− ·006	− ·132	−36	+ 5·7	+ 1	+32·4	+ 26	+14·7	− 57
4421	6492	d ,,	5	4·37	260	15	−29	47	+ ·018	− ·156	+37·1	− 5·4	+ 15	−31·7	+ 35	−18·4	− 64
4423	6493	— ,,	o	4·61	260	20	− 5	0	− ·093	− ·049	o	0·0	− 43	0·0	+ 9	0·0	− 23
4457	6553	θ Scorpii	o	2·04	262	32	−42	56	+ ·004	− ·011	+ 5	− 0·5	+ 3	− 3·6	+ 3	− 3·4	− 4
4466	6569	λ Aræ	5	4·84	263	10	−49	21	+ ·108	− ·179	+ 3·4	− 0·3	+ 59	− 2·2	+ 58	− 2·6	− 55
4481	6595	58 Oph.	5	4·89	264	22	−21	38	− ·089	− ·054	+11·5	− 1·1	− 41	· 10·6	+ 14	− 4·2	− 24
4483	6596	ω Dracon.	5	4·87	264	23	+68	48	+ ·010	+ ·327	−13·7	+ 0·5	+ 19	+ 4·9	+143	−12·8	+ 56
4492	6615	ι_1 Scorpii	5p	3·14	265	9	−40	5	+ ·003	+ ·002	−27·8	+ 1·8	+ 2	+21·2	− 1	+17·9	+ 1
4493	6616	X Sagittar.	8p	$\frac{4\cdot35}{5\cdot02}$ var.	265	19	−27	48	− ·005	− ·022	−13·5	+ 1·0	− 2	+11·9	+ 5	+ 6·3	− 9
4504	6636	ψ Dracon.	5	4·90	265	56	+72	12	+ ·015	− ·267	−10·4	+ 0·2	− 2	+ 3·2	−121	− 9·9	− 39
4539	6701	35 ,,	5	5·04	268	29	+76	59	+ ·048	+ ·239	−22·7	+ 0·1	+ 25	+ 5·1	+110	−22·1	+ 26
4542	6707	ν Herc.	op	4·48	268	40	+30	12	+ ·001	+ ·004	−21·6	+ 0·4	+ 1	+18·7	+ 1	−10·9	+ 2
4544	6710	ζ Serp.	o	4·60	268	48	− 3	41	+ ·144	− ·052	−40	+ 0·8	+ 68	+39·9	+ 13	+24·2	− 25

(1) P.G C.	(2) R.H.P.	(3) Star.	(4) Sp. F.	(5) Mag.	(6) R.A.	(7) Decl.	(8) μ_1.	(9) μ_2.	(10) V.	(11) A.	(12) *a*.	(13) B.	(14) *b*.	(15) C.	(16) *c*.
					° ′	° ′	″	″							
4559	6733–4	r Oph.	o	{6·04 5·34}	259 25	− 8 11	+ ·021	− ·040	−40	+ 0·4	+ 10	+39·6	+ 3	+ 5·7	− 19
4564	6742	W Sagittar.	5*p*	$\frac{4\cdot29}{5\cdot14}$ var.	259 39	−29 35	+ ·009	− ·012	−28·6	+ 0·2	+ 4	+24·9	+ 3	+14·1	− 5
4582	6775	*b* Herc.	8	5·21	270 48	+30 33	− ·094	+ ·063	+ 0·4	0·0	− 45	− 0·3	+ 15	+ 0·2	+ 26
4623	6850	36 Dracon.	5	5·03	273 20	+64 22	+ ·345	+ ·029	−35 7	− 0·9	+163	+15·4	+ 22	−32·2	+ 6
4672	6927	χ ,,	8	3·69	275 43	+72 41	+ ·521	− ·369	+32·4	+ 1·0	+262	− 9·6	−141	+30·9	− 52
4707	6978	*d* ,,	8*p*	4·95	277 43	+56 58	− ·004	− ·010	−10·4	− 0·8	− 1	+ 5·6	− 4	− 8·7	− 3
4731	7020	δ Scuti.	o	4·74	279 12	− 9 9	+ ·013	− ·004	−48	− 7·6	+ 6	+46·8	+ 1	+ 7·6	− 2
4752–4	7056–7	ζ Lyræ	{−7 −7}	$\frac{4\cdot29}{5\cdot87}$	280 20	+37 30	+ ·023	+ ·014	−26·0	− 3·7	+ 10	+20·3	+ 6	−15·8	+ 5
4753	7061	110 Herc.	5	4·26	280 20	+20 27	− ·021	− ·344	+22·6	+ 3·8	0	−20·8	− 58	+ 7·9	−153
4851	7226–7	γ Cor. Aus.	8	{5·01 5·01}	284 55	−37 12	+ ·094	− ·286	−51·0	−10·5	+ 22	+39·2	+ 91	+30·8	−108
4872	7261	17 Lyræ	o	5·04	285 55	+32 21	+ ·120	+ ·013	+ 4	+ 0·9	+ 54	− 3·2	+ 19	+ 2·1	+ 5
4874	7264	π Sagittar.	2*p*	3·02	285 57	−21 11	− ·006	− ·040	−10·5	− 2·7	− 4	+ 9·4	+ 6	+ 3·8	− 18
4891	7292	ψ ,,	5	4·93	287 21	−25 26	+ ·041	− ·035	−30	− 3·1	+ 16	+25·9	+ 13	+12·9	− 15
4929	7337.	β_1 ,,	o	4·24	288 52	−44 39	− ·001	− ·019	− 4	− 0·9	− 3	+ 2·7	+ 6	+ 2·8	− 6
4932	7340	ρ_1 ,,	−5	3·95	288 58	−18 2	− ·021	+ ·020	+ 2	+ 0·6	− 10	− 1·8	− 6	− 0·6	+ 9

4933	7343	β_2 ,,	0	4·51	289	0	−44	59	+ ·108	− ·063	+24	+ 5·5	+ 42	−16·1	+ 37	−17·0	− 21
4962	7387	ν Aquilæ	0	4·87	290	21	+ 0	8	− ·010	+ ·005	− 1·7	− 0·6	− 5	+ 1·6	− 2	0·0	+ 2
5014	7469	θ Cygni	5	4·64	293	26	+49	59	− ·028	+ ·247	−27·7	− 7·1	− 48	+ 16·3	+ 77	−21·2	+ 75
...	7518	SU ,,	5	$\frac{6\cdot2}{7\cdot0}$ var.	295	12	+29	1	...	...	−33	−12·3	...	+26·1	...	−16·0	...
5229	7796	γ ,,	8p	2·32	304	40	+39	56	+ ·001	− ·003	− 5·1	− 2·2	+ 1	+ 3·2	0	− 3·3	− 1
5244	7822	ρ Capric.	0	4·96	305	47	18	9	− ·014	·022	+21	+11·7	− 7	− 16·2	− 1	− 6·5	− 10
5255	7834	41 Cygni	5p	4·09	306	20	+30	2	+ ·009	− ·004	−18·4	− 9·4	+ 4	+12·8	+ 2	− 9·2	− 2
5274	7859	ρ Pav.	5	5·03	307	18	−61	52	+ ·047	− ·063	+ 7·1	+ 2·0	+ 2	− 2·7	+ 35	− 6·3	− 14
5286	7875	ϕ_2 ,,	8	5·30	307	56	−60	53	+ ·299	− ·573	−30·6	− 9·1	− 34	+ 11·8	+274	+26·7	−132
5291	7882	β Delph.	5	3·72	308	13	+14	15	+ ·108	− ·037	−25	−15·0	+ 43	+19·0	+ 28	− 6·2	− 17
5328	7936	ψ Capric.	8	4·26	310	3	−25	38	− ·057	− ·159	+26·5	+15·4	− 42	−18·3	+ 8	−11·5	− 68
5332	7943	ι Micr.	0	5·14	310	26	−44	21	+ ·167	− ·097	−19	− 8·8	+ 39	+10·3	+ 76	+13·3	− 33
5370	7988	T Vulpec.	0	$\frac{5\cdot53}{6\cdot13}$ var.	311	48	+27	53	+ ·007	− ·013	− 1·3	− 0·8	+ 4	+ 0·9	0	− 0·6	− 5
5371	7990	μ Aquar.	−7	4·80	311	49	− 9	22	+ ·037	− ·035	− 7	− 4·6	+ 11	+ 5·1	+ 14	+ 1·1	− 16
5390	8021	α Octant.	2	5·24	313	9	−77	24	− ·009	− ·362	+76	+11·3	−118	−12·1	+119	−74·2	37
5411	8048	ζ Micr.	0	5·35	314	9	−39	1	− ·020	− ·120	+ 7	+ 3·8	− 32	− 3·9	+ 19	− 4·4	− 44
5443	8097	γ Equul.	0p	4·76	316	22	+ 9	44	+ ·053	− ·161	−16·0	−11·4	+ 27	+10·9	+ 9	− 2·7	− 75
5455	8123	δ ,,	5	4·61	317	24	+ 9	36	+ ·041	− ·303	− 14	−10·2	+ 31	+ 9·3	− 2	− 2·3	−142
5493	8181	γ Pav.	8	4·30	319	33	−65	49	+ ·082	+ ·811	−31·1	9·7	+292	+ 8·3	−198	+28·4	+158
5562	8278	γ Capric.	0p	3·80	323	38	−17	7	+ ·187	·021	− 31	23·9	+ 50	+17·6	+ 73	+ 9·1	− 10

(1) P.G.C.	(2) R.H.P.	(3) Star.	(4) Sp. F.	(5) Mag.	(6) R.A.	(7) Decl.	(8) μ_1.	(9) μ_2.	(10) V.	(11) A.	(12) *a*.	(13) B.	(14) *b*.	(15) C.	(16) *c*.
					° ′	° ′	″	″							
5587	8309	μ_1 Cygni	5	4·73	324 55	+28 17	+ ·283	− ·240	+18·8	+13·6	+121	− 9·5	+ 79	+ 8·9	−100
5592	8315	κ Pegasi	5	4·27	325 2	+25 11	+ ·033	+ ·002	− 5	− 3·7	+ 9	+ 2·6	+ 13	− 2·1	+ 1
5623	8351	μ Capric.	0	5·18	326 58	−14 1	+ ·307	+ ·009	−17	−13·8	+ 80	+ 9·0	+121	+ 4·1	+ 4
5688	8430	ι Pegasi	5	3·96	330 35	+24 51	+ ·299	+ ·018	− 4·1	− 3·2	+ 67	+ 1·8	+125	− 1·7	+ 8
5698	8447	τ Pisc. Aus.	8	5·06	331 4	−33 2	+ ·446	+ ·006	−15·3	−11·2	+104	+ 6·2	+184	+ 8·3	+ 2
5709	8454	π Pegasi	5	4·38	331 23	+32 41	− ·013	− ·023	− 7	− 5·2	+ 2	+ 2·8	− 8	− 3·8	− 9
5758	8515	ν Indi	10	5·42	334 0	−72 44	+1·275	− ·694	+25·6	+ 6·8	− 18	− 3·3	+681	−24·4	− 98
5793·4	8558–9	ζ Aquarii	5	{4·59 4·42}	335 55	− 0 32	+ ·190	+ ·031	+27·1	+24·7	+ 37	−11·1	+ 82	− 0·3	+ 15
5874	8665	ξ Pegasi	5	4·31	340 26	+11 40	+ ·226	− ·496	− 4·6	− 4·2	+ 81	+ 1·5	+ 85	− 0·9	−230
5894	8697	σ ,,	0	5·30	341 50	+ 9 18	+ ·514	+ ·043	+13·4	+12·6	+ 73	− 4·1	+232	+ 2·2	+ 20
5949	8787	θ Gruis	5	4·35	345 19	−44 4	− ·046	− ·034	+10·0	+ 6·9	− 16	− 1·8	− 18	− 7·0	− 12
5975	8830	7 Androm.	0	4·62	347 0	+48 52	+ ·093	+ ·096	+12·8	+ 8·2	− 23	− 1·9	+ 51	+ 9·6	+ 30
6024	8905	υ Pegasi	10	4·57	350 6	+22 51	+ ·189	+ ·028	−12	−10·9	+ 10	+ 1·9	+ 89	− 4·7	+ 12
6077	8969	ι Piscium	10	4·28	353 42	+ 5 5	+ ·371	− ·439	+ 6·0	+ 5·9	+ 38	− 0·7	+172	+ 0·5	−207
6135	9045	ρ Cassiop.	8*p*	4·85	357 21	+56 57	− ·005	+ ·005	−42·1	−22·9	− 2	+ 1·1	− 2	−35·3	+ 1
6156	9072	ω Piscium	5	4·03	358 33	+ 6 19	+ ·151	− ·109	− 2·4	− 2·4	+ 7	+ 0·1	+ 71	− 0·3	− 51

The Astrographic Magnitude Scales. Ninth Note. The Toulouse Magnitudes: the Cape Magnitudes: and Further Remarks on the Obscured Patch as a Spiral. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The eighth note, presented last month, gave a provisional indication of the spiral form of the obscured patch described in April (see diagram on p. 480 of vol. lxxv.), and it was expected that several months would elapse before a sensible advance could be made on that indication. But the receipt of a kind present from Mr. Hough has quite altered the aspect of the matter, and I hasten to show appreciation of it by using it at once. He has sent me restricted counts (for two points in the Cape scale) for all the zones $-44°$ to $-51°$, which, with the figures already published for $-41°$ and $-42°$, and those for $-43°$, obtained by the energy of Mr. and Miss Bellamy from an advance copy of the volume kindly sent by the Astronomer Royal, give restricted counts for the whole of the Cape zones from $-41°$ to $-51°$—a most valuable mass of material, as will be seen below. These restricted counts, though very valuable for studying the obscured patches in the sky and the galactic condensation, problems which have arisen in the course of this discussion, do not advance the original inquiry as to the co-ordination of the various magnitude scales, which must not be lost sight of. Hence the opportunity will first be taken to make another contribution to that original inquiry by tabulating the counts of zone $+9°$, taken at Toulouse. Counts of a few Toulouse plates were made in 1909 and given in the first of these notes (*Mon. Not.*, lxix. p. 399), but no complete zone has yet been given.

The Toulouse Results.

2. The Toulouse results for zone $+9°$ (as also some other zone) are published in two parts, the first containing results for 0^h–6^h, and the second for the rest of the circuit. There is a curious and rather disturbing difference between the two parts. In the first the magnitudes are given in steps of one-quarter of a magnitude, the faintest record being 12·3, and others being denoted 12·0, 11·8, 11·5, 11·2, 11·0, 10·8, 10·5, 10·2, 10·0, etc., so as to denote quarters to the nearest decimal. In the second part a scale of tenths is adopted, with fair success, as the counts show; but at the same time the scale is extended to 13·3, though magnitudes fainter than 13·0 are rare. I have not been able to find in the introductions to the volumes any explanation of this change. The counts have been made for every unit of the scales; but it will suffice to give the *cumulative* totals for the points shown in Table I.

[Table.

TABLE I.

Cumulative Totals (N′) for Toulouse +9° for each hour of R.A.

	8·0.	8·5.	9·0.	9·5.	10·0.	10·5.	11·0.	11·5.	12·0.	12·3.	13·3.
0h	9	16	36	57	100	168	294	438	613	703	...
1	14	19	38	56	98	141	269	414	545	626	...
2	35	40	61	77	120	163	276	484	647	754	...
3	20	29	49	64	107	146	277	510	753	906	...
4	45	54	82	116	190	258	440	703	1026	1236	...
5	54	68	126	195	375	497	794	1188	1680	2295	...
6	155	244	397	655	880	1114	1478	1991	2873	3785	4783
7	67	129	216	367	524	731	935	1314	1972	2743	3949
8	16	30	51	120	204	333	465	745	1086	1496	2082
9	16	25	40	91	131	239	344	547	814	1145	1583
10	12	25	54	107	143	215	277	378	503	672	1055
11	17	24	53	103	147	199	262	362	501	672	999
12	15	19	32	52	78	119	169	262	371	500	672
13	12	27	46	83	114	167	228	330	435	610	831
14	17	24	51	96	140	201	249	332	473	605	822
15	11	20	45	107	153	230	322	497	694	997	1448
16	36	63	113	227	306	398	537	683	953	1280	1981
17	36	62	112	233	312	456	610	897	1313	1876	2751
18	79	147	294	596	799	1085	1455	1983	2693	3654	5214
19	65	114	200	395	521	706	992	1422	2088	2927	4096
20	114	174	296	496	631	892	1184	1533	2055	2705	3593
21	39	55	92	170	247	352	475	703	982	1386	1741
22	42	72	119	199	274	376	484	679	933	1257	1557
23	31	51	84	137	192	279	385	521	695	870	1103
N′	957	1531	2687	4799	6786	9465	13201	18916	26698	35700	40260
log N_0	0·726	0·930	1·174	1·426	1·576	1·721	1·866	2·022	2·171	2·298	2·350
Mag. C. & M.	8·49	8·92	9·46	10·03	10·37	10·73	11·09	11·49	11·89	12·23	(12·38)

3. At the feet of the columns are given the cumulative totals for the whole circuit of 180 plates (N′), and then $\log N_0 = \log N' - \log 180$, the values for a single plate. From the extended table (given on p. 145 of vol. lxxv.*) of Chapman and Melotte, the magnitudes in the last line are deduced. It will be seen that there is a curious jump near 10·5: up to 9·5 the scale is in accordance with that of C. and M., then the next

TABLE II.

Toulouse Magnitude Scale.

Toulouse.	C. & M.	Diff.	Toulouse.	C. & M.	Diff.	Toulouse.	C. & M.	Diff.
8·0	8·49	−0·49	9·5	10·03	−0·53	11·5	11·49	+0·01
8·5	8·92	−0·42	10·0	10·37	−0·37	12·0	11·89	+0·11
9·0	9·46	−0·46	10·5	10·73	−0·23	12·3	12·23	+0·07
			11·0	11·09	−0·09	13·3	(12·38)	(+0·92)

* Erratum in this table: for 13·1 the number should be 2·599 instead of 2·589 as printed.

whole Toulouse magnitude and a half is only 1·0 on the C. and M. scale; the Toulouse scale then recovers and remains nearly correct up to 12·3. The last figures must be bracketed as not interpretable at present, since there are apparently no corresponding stars in the first six hours.

4. For discussion of the variation in relative frequency of bright and faint stars, the points denoted 9·0 and 12·0 on the Toulouse scale were selected. The range in C. and M. magnitude is thus $11\cdot89 - 9\cdot46 = 2\cdot43$. In Table III. are given the excesses of the differences of log N for each hour over the mean value 1·06. The logarithms at the feet of the columns for 9·0 and 12·0 in Table I. differ by 1·00, which, being formed from sums of numbers, differs from the 1·06 formed from means of logarithms.

TABLE III.

Variations in log $N_{12\cdot0} - \log N_{9\cdot0}$ *(Toulouse).*

(The unit is 0·01.)

0^h	+17	6^h	−20	12^h	0	18^h	−10
1^h	+10	7^h	−10	13^h	− 8	19^h	− 4
2^h	− 4	8^h	+27	14^h	−10	20^h	−22
3^h	+13	9^h	+25	15^h	+13	21^h	− 3
4^h	+ 4	10^h	− 9	16^h	−13	22^h	−17
5^h	+ 7	11^h	− 8	17^h	+ 1	23^h	−14

5. The portion 0^h to 5^h is positive; the rest of the circuit is negative, with three exceptions.

Forming the first harmonic (for reasons explained in discussing the Cape results below) we get

$$+ \cdot065 \sin\alpha - \cdot010 \cos\alpha = \cdot066 \cos(\alpha - 99^\circ).$$

This is for a magnitude difference of 2·43 as above. Dividing the coefficient by this, the mean value per magnitude is thus

$$+ \cdot027 \cos(\alpha - 99^\circ) = - \cdot027 \cos(\psi - 65^\circ).$$

For the meaning of ψ see below.

The Cape Counts.

6. In the Cape volumes diameters are measured and given in figures for the brighter stars, but estimates were made for the fainter, and are printed with a negative sign preceding. Hence counts can be rapidly made for two points of the scale, viz., all the stars measured (M) and the total number (T) on the plates measured + estimated. The values of log T − log M are now our chief concern. Details for zone − 41° will be found on p. 59 of vol. lxxv., and the values there found for the whole circuit are

$$\log T - \log M = 4\cdot854 - 4\cdot440 = 0\cdot454,$$

corresponding to a magnitude difference on the C. and M. scale (given on p. 145, vol. lxxv.) of

$$13{\cdot}45 - 12{\cdot}22 = 1{\cdot}23.$$

The corresponding log ratio for other zones is

−41°	·454	−47°	·437
−42	·405	−48	·423
−43	·404	−49	·406
−44	·442	−50	·411
−45	·414	−51	·424
−46	·407	Mean	·421

We may adopt the mean value 0·42 as applicable to all the zones.

7. From this mean value there are sensible variations as we travel round the R.A. circuit. These differences are given in Table IV., but they have been rearranged, in a manner which must now be explained (it seems scarcely worth while to print them twice over).

8. In the last note it was stated that the obscured patches on the sky form a spiral the median line of which is approximately

$$\alpha + 4\delta = 260^\circ.$$

Points on the median line for different zones were taken as the middle of the "obscured patch" for that zone which had been assigned in the notes already published. But this is a somewhat arbitrary process, especially when the "obscured patch" is not continuous but is broken up into two or more; it is desirable to substitute some automatic process; and after several trials it was decided simply to calculate the first harmonic and take the minimum (representing greatest "obscurity"). The reason for previous hesitation about the applicability of this process was that the numbers suggested discontinuities or breaks rather than sinuous curves; but on consideration this reason was over-ruled.

9. The curve deduced in this way, as shown below, from all available material, is

$$\alpha + 3{\cdot}66\delta = 247^\circ.$$

It has not as yet been followed beyond the limits $\delta = +62^\circ$ and $\delta = -51^\circ$, but this difference in δ corresponds to 414° in α, *i.e.* more than one complete circuit, which justifies the name spiral. Let us write

$$\psi = \alpha + 3{\cdot}66\delta - 247^\circ,$$

so that $\psi = 0$ is the equation to the median line of the spiral. [The symbol ψ is adopted as suggesting the first two letters of "spiral," inverted, it is true, but there is no single symbol for them as they stand.]

10. Then in Table IV. the differences of log T − log M from its mean value have been arranged so as to put corresponding values of ψ together as nearly as possible. The usual order can easily be recovered if wanted by rewriting, noticing that the first

number is for the R.A. given at the top of the column. The reference numbers in the first column are thus values of ψ counted in hours.

The mean values in the last column have been divided by the difference of magnitude 1·23, to get the mean per magnitude. They show a pretty steady rise up to (15) and then a sudden drop in 4 hours; indeed, the greater portion of the drop takes place in 2 hours.

TABLE IV.

Cape Zones. Variations of log T − log M from 0·42.

(The unit is 0·01.)

Zone. ψ_0.	−41°. 2h.	−42°. 3h.	−43°. 3h.	−44°. 3h.	−45°. 3h.	−46°. 4h.	−47°. 4h.	−48°. 4h.	−49°. 4h.	−50°. 5h.	−51°. 5h.	Mean per Mag.
(0)	+ 5	−10	+ 1	− 7	− 9	− 4	− 4	− 3	− 6	− 6	+ 2	−3·0
(1)	+ 2	−13	+ 7	+ 3	0	− 6	− 2	− 6	0	+ 2	− 3	−1·2
(2)	− 3	−11	0	− 3	− 1	+ 1	− 6	− 9	− 2	+ 1	+ 2	−2·3
(3)	− 5	− 5	− 1	+ 6	− 1	+ 4	− 1	+ 3	−14	− 2	+ 6	−0·7
(4)	− 2	− 6	+ 5	+ 3	+ 8	− 5	− 6	−10	− 4	+ 6	+ 7	−0·3
(5)	− 4	− 9	− 6	− 3	+ 1	− 5	+ 1	− 5	−10	− 2	− 1	−3·1
(6)	−10	− 8	− 4	− 3	− 3	+ 5	+ 9	+ 7	− 6	− 3	− 1	−1·3
(7)	+ 5	− 4	+ 2	+ 1	+ 9	0	+ 5	+ 9	− 5	− 3	+ 1	+1·5
(8)	+ 5	+10	0	+ 4	+ 7	+ 9	− 1	+12	− 7	− 1	− 6	+2·3
(9)	+ 7	+ 5	+ 6	+ 4	− 6	+ 4	+ 9	− 2	− 2	− 2	+ 3	+1·9
(10)	+ 1	+ 4	+10	+ 7	− 8	+ 9	+10	+ 4	+ 4	− 4	0	+2·7
(11)	+11	+10	+ 6	+ 7	− 5	+ 4	+20	+13	+ 2	− 3	+ 9	+5·4
(12)	+16	+12	+ 4	− 2	− 3	+11	+19	+16	+ 2	+ 4	+18	+7·2
(13)	+18	+ 5	+ 9	+ 8	+ 2	−13	+ 8	+ 8	+ 1	+13	− 8	+3·8
(14)	+15	+12	+ 9	+ 9	+ 8	+ 5	+14	+11	+14	+ 6	+ 5	+7·8
(15)	+12	+11	− 7	+27	+24	+ 4	+32	+10	+11	− 2	− 4	+8·8
(16)	− 2	+10	−11	+11	+11	0	+ 4	+ 6	+ 9	+ 2	− 7	+2·4
(17)	− 6	+ 2	−12	− 5	+ 1	− 3	− 7	− 3	+ 4	− 6	−16	−3·8
(18)	+ 1	+ 3	−17	− 2	+17	− 3	−15	− 5	+ 1	− 7	− 8	−2·6
(19)	+ 5	−15	−20	− 6	−17	− 2	− 7	−17	− 2	− 5	+ 2	−6·2
(20)	+ 2	− 6	−14	− 5	−15	−11	−14	− 9	− 9	− 7	+ 1	−6·3
(21)	− 6	−11	− 5	− 1	+ 1	−13	− 5	− 7	− 6	− 4	+ 2	−4·0
(22)	− 6	−19	− 1	− 9	−10	−14	− 7	− 9	− 8	− 5	− 1	−6·5
(23)	0	− 4	+ 1	− 1	−23	− 5	−12	− 6	+ 1	− 5	+ 7	−3·4

11. We must now explain more fully the basis of this arrangement. In previous notes the counts for a number of different observatories and different zones have been given. Attention has been called to the variations in the log ratio, but no harmonic analysis has been applied, except in one or two of the earlier papers, when a seasonal cause was suspected. Some discussion of the possibility of this cause was given in the case of the Bordeaux results (*M.N.*, lxxii. pp. 470–1), the conclusion being against any temperature effect. Further experience is in the same direction.

If the effect were seasonal it should be approximately the same (say) at Rome and Algiers, whereas it is in opposite directions, as the phases in Table V. show.

12. Expressing the log ratios in the form

$$C \cos (\alpha - A),$$

the log ratio is a maximum when $\alpha = A$. But we have hitherto used the middle of the *obscured* patch as reference point, which makes the minimum more suitable than the maximum. Hence we reverse the sign of C and write

$$C \cos (\alpha - A) = - C \cos (\psi - \psi_0),$$

where $$\psi_0 = A + 3{\cdot}66\delta - 247^\circ.$$

13. The coefficient 3·66 was deduced from a study of the data in Table V., as grouped in the last column in sets of 3 zones.

TABLE V.

Observatory.	Zone.	A.	ψ_0.	Mean ψ_0.
	°	°	°	°
Vatican	+64			
Vatican	+62	14	− 6	− 6
Vatican	+60			
Oxford	+31	140	+ 6	
Oxford	+30	167	+ 30	
Oxford	+29	125	− 16	+11
Oxford	+28	273	+128	
Oxford	+26	137	− 15	
Paris	+22	211	+ 45	+ 4
Bordeaux	+17	180	− 5	
Toulouse	+ 9	279	+ 65	
Algiers	0	215	− 32	− 1
Algiers	− 1	215	− 36	
Perth	−32	38	+ 34	
Cape	−41	50	+ 13	+17
Cape	−42	69	+ 28	
Cape	−43	345	− 59	
Cape	−44	58	+ 10	−23
Cape	−45	51	− 1	
Cape	−46	29	− 26	
Cape	−47	53	− 6	−13
Cape	−48	49	− 14	
Cape	−49	105	+ 39	
Cape	−50	55	− 15	+ 2
Cape	−51	341	− 93	

14. There are of course Greenwich results available above $+64°$; but knowing how thorough an investigation is being made at the Observatory itself, I have hesitated to intrude on this domain.

15. The three Vatican zones were not treated separately, but were combined (*M.N.*, lxxv. p. 601). The Paris figures have not yet been published, but are in MS. at Oxford, as also the figures for Algiers 0°. In both these cases restricted counts have been made as for the Cape, and the essential facts are given in Table VI. below. The results for Algiers $-1°$ are those published in *M.N.*, lxxii. p. 700, under the heading $+1°$, by an unfortunate mistake.

16. The second column gives the declination of the plate centre, and the third column the value of A in the formula

$$C \cos (\alpha - A),$$

deduced by combining 23^h and 0^h, 1^h and 2^h, etc., so as to form twelve terms, which were submitted to the ordinary process of analysis.

The value of ψ_0 in the fourth column is found by the formula given in § 13.

17. The mean values of ψ_0 in the last column were formed from the sines and cosines of the individual values bracketed, and not from the phases, *i.e.* the coefficients as well as the phases came into the operation. In the cases of large deviation, especially Oxford $+28°$, $\psi_0 = +128°$, and Cape $-51°$, $\psi_0 = -93°$, the coefficients C were small, so that these discordances did not affect the mean of the group very seriously.

18. The mean values of ψ_0 in the last column are the practical justification for the hypothesis of the spiral. The figures were obtained by making these values as near zero as possible, and for this kind of investigation they are surprisingly small.

19. The results in the southern hemisphere (with the exception of Perth, which was omitted in order to form a homogeneous mean) have been grouped according to ψ in Table IV. We will now do the same for the northern hemisphere in Table VI. We will omit the Vatican results for the present, as they are somewhat distant from the rest, and further, the three zones have not yet been separated. The differences of magnitude were larger for these zones than for the Cape. [The small difference for the Cape was adopted purely for convenience of rapid counting: a larger value would have been better.] They were as follows on the C. and M. scale:—

Oxford.	Paris.	Toulouse.	Bordeaux.	Algiers.
3·3	1·9	2·4	2·5	1·4

The mean may be taken as 2·5, which has been used as divisor for the last column.

20. Now the striking difference between Table VI. and Table IV. is that the sudden change is now on the other side of the maximum: *i.e.* it is now a sudden *rise* from (4) to (9)

instead of a sudden *fall* from (15) to (19). In terms of the hypothesis of an "obscured patch," the patch is only sharply defined on one side, shading gradually away on the other; and this is the case in both hemispheres, but the sharp side in the northern hemisphere is of smaller R.A., or rather smaller ψ; in the southern hemisphere of greater ψ.

TABLE VI.

For the Northern Hemisphere; similar to Table IV. for Southern.

(The unit is 0·01.)

Zone. ψ_0.	+31°. 9h.	+30°. 9h.	+29°. 9h.	+28°. 10h.	+26°. 10h.	+22°. 11h.	+17°. 12h.	+9°. 14h.	0°. 16h.	−1°. 17h.	Mean per Mag.
(0)	−15	−17	−10	− 5	+ 2	−17	−14	−10	− 1	− 6	−3·7
(1)	− 8	− 4	−10	− 7	+ 1	− 1	−14	+13	− 1	−18	−2·0
(2)	− 3	−13	−36	+ 3	−28	−33	−30	−13	−10	+ 8	−6·2
(3)	−13	− 8	− 7	−12	−12	−24	− 2	+ 1	+ 2	+14	−2·4
(4)	− 6	−23	− 2	−12	−19	−11	− 4	−10	+ 5	+10	−2·9
(5)	0	− 9	+20	− 2	− 4	− 6	−27	− 4	+ 4	−27	−2·2
(6)	− 7	− 8	− 4	− 5	+18	+ 2	+ 1	−22	− 1	+ 9	−0·7
(7)	+ 6	+ 6	− 5	−10	+25	+ 8	+ 5	− 3	− 5	+ 5	+1·3
(8)	+ 1	+13	+ 4	−14	+17	− 3	+13	−17	+ 8	+ 8	+1·2
(9)	+22	+ 9	+13	+16	− 1	+30	+39	−14	+11	+ 8	+5·3
(10)	+21	+10	+ 1	+ 8	+ 9	+14	+33	+17	0	+12	+5·0
(11)	+ 8	− 4	+12	+24	+ 9	+12	+22	+10	+ 5	− 2	+3·8
(12)	+ 9	+ 9	+ 1	+16	+13	− 3	+ 6	− 4	+ 5	−33	+0·8
(13)	+11	+ 7	+ 8	+ 7	− 9	−11	+ 4	+13	− 4	+ 9	+1·4
(14)	+ 2	+30	+14	− 8	+11	−26	+11	+ 4	+ 5	+30	+2·9
(15)	+19	− 1	−12	−13	+25	+ 1	+16	+ 7	− 7	+11	+1·8
(16)	+ 5	+ 3	−16	−25	+ 9	0	− 4	−20	+21	+12	−0·6
(17)	+ 4	− 2	− 5	−20	−15	+15	+28	−10	+ 3	−11	−0·5
(18)	− 9	− 8	− 3	−19	−13	+38	+33	+27	− 9	−17	+0·8
(19)	+ 3	+ 6	−10	+10	−15	+15	− 8	+25	−16	−20	−0·4
(20)	+ 3	− 6	− 9	+30	− 3	+12	−18	− 9	− 8	− 2	−0·4
(21)	−14	+ 5	+13	+37	− 8	+ 8	−28	− 8	− 8	+ 2	0·0
(22)	− 5	+ 9	+27	− 8	− 3	−12	−33	0	− 9	−16	−2·0
(23)	− 9	− 2	− 4	+14	−15	−11	−24	− 8	−11	+ 8	−2·5

21. The process of adding together the figures for a number of zones must tend to obliterate such a feature as a sharp edge, unless it runs strictly parallel to the median line of the spiral, which is unlikely. It is the more remarkable that so much sharpness has been preserved. When we have more material to elucidate this phenomenon, we may be able to devise a better procedure than that of calculating the first harmonic, which is a rough-and-ready procedure scarcely suited to the character made manifest above.

There is a sharp peak in the curves for both hemispheres, and it may be that by bringing these peaks together we can get a better equation to the spiral; but this is a refinement scarcely justified at present.

22. An appeal may here be made for more counts, especially in the gap -1° to -30°. If the spiral really changes one edge for the other, it would seem to be in this gap. San Fernando, Tacubaya, Hyderabad, Santiago, Cordoba could probably throw much light on this interesting problem by publishing the measures of a complete zone, or having counts made.

Summary.

(1) The spiral

$$\alpha + 3{\cdot}66\delta = 247^\circ,$$

runs centrally through a region where the ratio of the number of (say) 12·0 magnitude stars to the number of (say) 10·0 magnitude is low, with an accuracy shown by the residuals in Table V.

(2) This region of "obscuration" is bounded sharply on the side of smaller R.A. in the northern hemisphere (see Table VI., last column), and sharply on the side of greater R.A. in the southern (see Table IV., last column).

(3) It is impossible at present to specify where the change takes place from one side to the other, as no results have been published between -1° and -31°.

[*Note added* 1916 *Jan.* 2.—The evidence at present available that the phenomenon is not a seasonal effect may be briefly summarised as follows:—If we add together corresponding R.A.'s and form the first harmonic, we get

$$\text{For the N. hemisphere} + 2{\cdot}7 \cos(\alpha - 359^\circ).$$
$$\text{For the S. hemisphere} - 5{\cdot}0 \cos(\alpha - 50^\circ).$$

The coefficient for the S. hemisphere has been reversed so as to show corresponding phases; and it will be seen that they differ by 51°, which seems too large to be accidental. Moreover, the Vatican results, not included in the above analysis, would be discordant by 165°.

The corresponding figures for the hypothesis shown in Tables IV. and VI. are

$$\text{For the N. hemisphere} + 3{\cdot}2 \cos(\psi - 190^\circ).$$
$$\text{For the S. hemisphere} + 6{\cdot}0 \cos(\psi - 177^\circ).$$

The coefficients are increased—not very much, it is true; but the range of ψ in each group is not large, and we cannot expect a large increase; the phases are brought into accord, and the omitted Vatican results are also in accord.]

Baxendell's Observations of Variable Stars. Fifth Instalment. No. 13, T Herculis; No. 14, R Leonis; No. 15, S Orionis. Edited by H. H. Turner and Mary A. Blagg.

No. 13, *T Herculis* (6512 *Chandler*).

$18^h\ 3^m\ 37^s\ +30^\circ\ 59'{\cdot}9$ (1855·0).
$18\ \ 5\ \ 19\ \ +31\ \ \ 0{\cdot}2$ (1900·0).

Identification of Comparison Stars.—In this case Baxendell appears to have kept the same designations for the comparison stars throughout. There is one chart, dated Nov. 7, 1858, on which the stars are named as below, except that *l*, *m*, *x*, *y*, and the two "companions" are not given. There is also one small diagram, 1886 Nov. 30, in which only T and *k* are named, and another, 1863 May 23, in which *x* and *y* are named. The relative positions in the former suggest some doubt as to whether the *k* of the chart is really Baxendell's *k*.* If it is, he saw it very much brighter than the magnitude found by interpolation below. There are similar differences in the stars which he called *d* and *f*, and a still more striking one in the case of *b*. But his differences between his comparison stars, as shown in Table II., display such remarkable fluctuations as to suggest that several of these stars are themselves variable. Four of them are noted as being so by Baxendell (see Table I.). There is no clue to the identity of *l*, except his estimated magnitude; and the only clues for *m* are his statement that it has a companion *s.p.*, and another south and smaller, and his estimate of its magnitude at 12·3 or 12·4. His estimates of magnitude in the case of the other comparison stars seem to be more than a magnitude too faint in most cases. "Companion of T" is mentioned 1859 Sept. 28; it seems probable that *h* is meant, but it may be "*y*."

The Harvard magnitudes not in brackets are from *H.A.*, xxxvii. p. 172; those in ( ) are interpolated from the table on p. 200; [5·21] is from Revised Harvard Photometry, *H.A.*, vol. l.; {6·65} and {11·76} from an Oxford photograph.

Star *a* (Bax.) is A (or 104) Herculis, $18^h\ 8^m\ 8^s\ +31^\circ\ 23'{\cdot}1$.
,, *b* (Bax.) is *b* (or 99) Herculis, $18\ 3\ 14\ +30\ 32'{\cdot}8$.

* It seems possible that the *k* of the chart is his "*l*."

TABLE I.

Baxendell's Comparison Stars.

Baxendell.		A.S.V. 11.		Harvard.		Adopted Mag.
Name.	Estimate.	No.	Grade.	Letter.	Mag.	
a	7·5	2	...	...	{6·65}	64
b	7·8	1	...	...	[5·21]	69
c	7·7 *v.*	3	0	*a*	6·67	68
d	8·8	4	11	*c*	7·17	78
e	8·7 *v.*	6	19	*e*	7·82	77
f	9·6 *v.*	5	18	*f*	8·10	85
j	...	11	39	...	(8·85)	90
g	{10·0 or 10·1}	14	47	*m*	9·44	91
h	10·9 *v.*	22	65	*n*	9·82	98
k	10·7	23	68	...	(10·33)	96
l	11·8	?	?	...	...	103
m	{12·3 or 12·4}	...	...	...	...	108
x	...	31	84	...	(11·16)	...
y	...	29	79	*p*	10·97	110
comp. *k*	{13·0 or 13·1}	...	...	...	{11·76}	118

TABLE II.

Baxendell's Differences between his Comparison Stars.

(The unit is a step, converted in the last line into 0·1 mag.)

Years.	68–64.	69–68.	69–64.	77–68.	77–69.	78–77.	78–69.	78–68.	85–77.	91–85.	98–91.	98–96.	103–96.	108–103.
1858–9	...	$-4\cdot0_1$	$2\cdot5_1$	...	...	$-3\cdot0_1$	...	...	$0\cdot5_5$	$10\cdot0_2$	$1\cdot5_1$	...	...	...
1863–5	$3\cdot7_6$	$+0\cdot5_{10}$	$4\cdot0_{16}$	...	...	...	$11\cdot0_1$	$15\cdot0_1$	...	$3\cdot0_1$	$3\cdot5_1$	...	...	$5\cdot0_1$
1866–8	$3\cdot9_4$	$+0\cdot6_9$	$4\cdot7_{21}$	$5\cdot7_3$	$6\cdot9_8$	$+2\cdot0_1$	$10\cdot0_6$	$12\cdot2_2$	$9\cdot2_6$	$3\cdot0_4$	$10\cdot0_2$	$1\cdot1_4$	$6\cdot0_1$	$5\cdot3_3$
1878–80	$9\cdot0_1$	$+1\cdot0_5$	$5\cdot2_{14}$	$6\cdot8_3$	$6\cdot0_2$	$+2\cdot7_5$	$9\cdot5_2$	$10\cdot3_5$	$9\cdot4_5$	$6\cdot0_5$	$5\cdot8_2$	$2\cdot9_{13}$	$8\cdot1_4$	$4\cdot5_2$
1881–2	...	$-3\cdot0_2$	$5\cdot9_5$	$10\cdot0_1$	$9\cdot0_2$	...	...	...	$12\cdot0_1$	$6\cdot5_2$	...	$-2\cdot8_3$	...	$7\cdot0_1$
1883–6	...	$-2\cdot5_2$	$7\cdot8_8$	$9\cdot3_2$	$9\cdot8_4$	$+1\cdot5_3$	$18\cdot0_2$	$12\cdot0_1$	$6\cdot0_2$	$7\cdot2_4$	$12\cdot0_1$	$+2\cdot5_{23}$	$7\cdot8_3$	$6\cdot8_4$
Mean.	4·3	0·0	5·1	7·3	7·7	+1·6	11·5	11·0	5·8	6·0	6·9	+2·1	7·7	6·0
No. of Comparisons.	11	29	65	9	16	10	11	9	19	18	7	43	8	11
Reduced Mean.	3·6	0·0	$4\cdot5_{64}$*	6·3	6·6	$+1\cdot8_9$*	9·7	9·4	$7\cdot5_{14}$*	$4\cdot7_{16}$*	$6\cdot1_6$*	+1·8	6·6	5·2

* In these cases the observations of 1858–9 are omitted.

Cases have been omitted where the total number of comparisons was less than 5.

The reduced means are obtained by applying to each comparison the formula of reduction used in the previous instalments of this series.

The telescopes used were A 68 until the end of 1869, and E 80 from 1878, with the following exceptions:—On 1859 Aug. 18, *a* (22) 28, and 1859 Sept. 4 and 10, *a* (22) 39 were used. On 1863 Aug. 15, R (7) 39, and 1863 Sept. 19, and 1864 Apr. 25, C 39 were used. On 1878 July 31, A (6), and Sept. 13, E 125 were used.

TABLE III.

Baxendell's Observations of T Herculis.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1859			
Aug. 1	0258	77+5, 90−3, 85+3	86
2	0259	85+T=90−T	88
10	0267	85+T=90−T, 77+4	84
18	0275	85+2, 90−6	86
19	0276	77+3, 85+4, 91−5	85
24	0281	77+2	79
Sept. 4	0292	77+8, 85+8, 90−2	89
10	0298	90+0	90
28	0316	?98 − 2 or 110 − 2	?96 or ?108
1863			
May 23	1649	110+14	124
June 3	1660	Invisible <110	
12	1669	,, <120	
20	1677	,, <127	
25	1682	,, <125	
July 3	1690	,, <125	
Aug. 15	1733	64+2, 69−1, 68+0	67
20	1738	64−2, 69−3	64
24	1742	64−2	62
Sept. 5	1754	64−2	62
19	1768	64+5, 69+1	70
1863			
Oct. 9	1788	78+8, 85−3	84
Nov. 2	1812	108+0	108
29	1839	Glimpsed with difficulty ?+3	?113
Dec. 17	1857	Not seen <125	
1864			
Apr. 25	1987	,, <110	
May 7	1999	Glimpsed 132	
26	2018	108+6	114
July 13	2066	64+5, 69+0	69
16	2069	64+4, 69−1	68
19	2072	64+1, 69−3	66
29	2082	64−1, 69−4	64
Aug. 21	2105	78−3	75
23	2107	78−3	75
1865			
May 1	2358	103+3, 108−1	106
24	2381	91−1, 98−4	92
June 9	2397	69+3	72
11	2399	64+4, 69+1, 68+1	69
13	2401	64+2, 69+1, 68−1	68
17	2405	64+3, 69−1	68
20	2408	64+3, 69−2	67
28	2416	64+3, 69−1	68

TABLE III.—*Baxendell's Observations of T Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1865			
July 1	2419	64+2, 69−1	67
11	2429	64+4, 69+1, 68+1	69
21	2439	78+0	78
Sept. 7	2487	Not seen	
Oct. 7	2517	108+3	111
29	2539	91−2	89
Nov. 12	2553	* ?69+6	?75
15	2556	69+3, 78−6	72
26	2567	68+1, 69−1	68
1866			
Apr. 23	2715	69+4, 78−3	74
25	2717	69+3, 78−4	73
29	2721	69+3, 78−6	72
May 7	2729	69+1, 78−7	70
15	2737	64+2, 69−1, 78−9	68
17	2739	64+3, 69−1	68
20	2742	64+3, 69−1	68
22	2744	64+3, 69−1	68
26	2748	64+3, 69−1	68
June 10	2763	69+2, 78−7, 68+2	71
25	2778	78+4, 85−3	82
July 31	2814	108+0	108
Aug. 27	2841	108+1	109
31	2845	108+1	109
Sept. 8	2853	103+3, 108−3	106
15	2860	98+1, 96+3	99
17	2862	96−1, 98−1	96
24	2869	85−1	84
Oct. 1	2876	77−1, 85−4	80
6	2881	69+4, 77−3, 68+3	73
8	2883	69+1, 77−3	72
10	2885	68+0, 69−1, 77−3	70
1866			
Oct. 14	2889	64+3, 69−1, 68+1	68
19	2894	64+0, 68−3, 69−3	65
28	2903	64−1, 69−5	64
Nov. 3	2909	64+0, 69−3	65
6	2912	64+0, 69−4	64
11	2917	64+0, 69−5	64
19	2925	64+0, 69−3	65
25	2931	64+5, 69+1, 77−5	70
30	2936	69+2, 77−5	72
Dec. 16	2951	85+2, 91−2, 98−8	89
1867			
Apr. 11	3068	64+4, 69+1, 68+1	69
21	3078	64+4, 69+1, 68+0	69
May 11	3098	77+2, 85−5	80
14	3101	77+5, 85−2	82
21	3108	77+7, 85−1	84
July 24	3172	Glimpsed with difficulty.	
27	3175	118+2	120
Aug. 1	3180	118+0	118
6	3185	118+0	118
21	3200	96+3, 103−2	100
Sept. 2	3212	85+1, 91−2	88
5	3215	77+9, 85−2	84
10	3220	77+2, 85−9	78
16	3226	69+5, 77−3	74
24	3234	69+4, 77−3	74
Oct. 3	3243	69+1, 64+4	69
23	3263	64+6, 69+2	70
Nov. 1	3272	77+3	80
19	3290	96−4	92

* The query is B.'s.

TABLE III.—*Baxendell's Observations of T Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1868			
Apr. 11	3434	69+2, 77−3	72
May 11	3464	96+2	98
30	3483	108+1	109
June 4	3488	108+3	111
12	3496	108+7	115
15	3499	108+7	115
30	3514	108+2	110
July 11	3525	103−2	101
12	3526	98+3, 103−3	100
21	3535	91+3, 98−7	92
Sept. 4	3580	64+3, 69−1	68
6	3582	64+3, 69−1	68
11	3587	64+6, 69+1	70
Oct. 17	3623	96+1	97
Nov. 8	3645	108−2	106
15	3652	103+2, 108−3	105
1869			
Apr. 30	3818	108+6	114
June 8	3857	103+0	103
Oct. 26	3997	Not seen <133	
1878			
June 1	7137	103+0	103
10	7146	103−5, 98−2	97
23	7159	85+3, 91−3	88
28	7164	77+2, 85−3	80
July 8	7174	68+3, 78−5	72
19	7185	64+9, 68+3, 69+1	71
27	7193	69+1, 68+2	70
31	7197	69+3	72
Aug. 6	7203	69+3, 78−4, 68+6	73
8	7205	68+5, 78−4, 77−1	74
18	7215	77+7, 85−2, 78+2	82
21	7218	77+5, 85−3	82
1878			
Aug. 25	7222	85+0, 91−4	86
28	7225	85+3, 91−1	89
Sept. 2	7230	91+0, 96−3	92
9	7237	96−1	95
11	7239	98+0, 96+3	98
13	7241	96+2, 98−1	98
24	7252	108−2	106
Oct. 13	7271	108+6	114
17	7275	108+7	115
26	7284	108+3	111
Nov. 1	7290	108−1	107
4	7293	108−2	106
8	7297	98+0	98
28	7317	77+5, 85−7	80
Dec. 8	7327	69+3, 77−4, 78−6, 68+2	72
1879			
Mar. 31	7440	108+6	114
Aug. 8	7570	98+2, 103−2	100
11	7573	103+0, 98+3	102
14	7576	98+6, 108−2, 103+2	105
23	7585	108+3	111
30	7592	108+5	113
Sept. 2	7595	108+6	114
7	7600	108+7, min. vis. −3	115
13	7606	108+3	111
19	7612	108+3	111
23	7616	108+0	108
25	7618	108−2	106
29	7622	108−3	105
Oct. 4	7627	98+1, 103−3, 96+5, 108−7	100
15	7638	91+0	91
Nov. 1	7655	77+3, 85−3	81
5	7659	77+2	79
13	7667	68+3, 77−3	72

TABLE III.—*Baxendell's Observations of T Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1879			
Nov. 23	7677	64+3, 69−2	67
25	7679	64+3, 69−3	66
27	7681	64+3, 69−1	68
29	7683	64+3, 69−2	67
Dec. 3	7687	64+2, 69−2	66
1880			
Mar. 17	7792	91+0	91
Apr. 1	7807	85−1	84
11	7817	77−1	76
24	7830	69+1, 77−3	72
30	7836	64+3, 69−2	67
May 6	7842	69+2	71
9	7845	64+4, 69+0	68
13	7849	64+4, 69+0	68
16	7852	64+2, 69−1	67
20	7856	64+4, 69+3	70
24	7860	64+5, 69+2	70
28	7864	64+5, 69+2	70
June 3	7870	77−1	76
17	7884	85+1, 91−5	86
July 4	7901	103−2	101
20	7917	108+3	111
30	7927	108+1	109
Aug. 9	7937	108−1	107
11	7939	108−1	107
13	7941	108−2	106
24	7952	98−3, 96−2, 91+3	94
29	7957	91−1	90
Sept. 4	7963	91−3, 85+3	88
15	7974	78+1, 77+0	78
26	7985	64+1, 69−3	66
28	7987	64+0	64
Oct. 2	7991	64+0	64
9	7998	64−2	62
10	7999	64−1	63
20	8009	64−2	62
Oct. 29	8018	64+2, 69−1	67
Nov. 3	8023	69+2	71
21	8041	69+9, 85−2	80
26	8046	85+3, 91−4	88
Dec. 1	8051	91−1, 96−6	90
1881			
Mar. 28	8168	64−2	62
Apr. 1	8172	64+1, 69−2	66
17	8188	64+4, 69+0	68
May 4	8205	91−1	90
7	8208	91−2	89
16	8217	98+0, 96−1	96
20	8221	98+3, 96+0	98
29	8230	108+1	109
June 12	8244	108+3	111
July 8	8270	108+4	112
17	8279	103+0, 108−6	102
27	8289	96−1	95
Aug. 1	8294	96−2	94
24	8317	69+6, 77−2	75
31	8324	69+1	70
Sept. 14	8338	64+4, 69−3	67
19	8343	64+3, 69−3	66
Oct. 2	8356	77−3	74
15	8369	91−3	88
20	8374	*78+4, *77−3	?78 or 95
29	8383	98+2	100
Nov. 17	8402	108+4	112
Dec. 1	8416	108+6	114
19	8434	108+3	111
1882			
Mar. 18	8523	68+3, 91−10, 69+9, 85−5	78
Apr. 6	8542	91+1	92
10	8546	98−2	96

* ? Error for 91 and 98.

TABLE III.—*Baxendell's Observations of T Herculis*—continued.

Date.	J.D. 240 +.	Corrected Comparisons.	Deduced Mag.
1882			
Apr. 20	8556	98+5, 96+1	100
May 12	8578	108+4	112
14	8580	108+3	111
20	8586	108+6	114
28	8594	108+10	118
July 9	8636	85+0, 91−5	86
24	8651	64+5, 69+0	69
Aug. 19	8677	69+2	71
Sept. 4	8693	69+9, 77+3	79
9	8698	77+9, 85+0	86
1883			
Mar. 24	8894	103+4, 108−2	106
Apr. 8	8909	<108+4	<112
July 28	9020	85−3	82
Aug. 25	9048	103+2, 108−3	105
Oct. 2	9086	108+6	114
28	9112	98+2	100
1884			
Mar. 21	9257	108+3	111
25	9261	108+3	111
Apr. 7	9274	103+1, 108−7	102
12	9279	96+2, 103−7	97
May 4	9301	77−1	76
18	9315	69−2, 64+5	68
22	9319	64+5, 69−2	68
24	9321	64+5, 69−2	68
26	9323	64+3, 69−3	66
30	9327	64+1, 69−5	64
June 4	9332	64+5, 69−3	68
11	9339	69+3	72
20	9348	77−3, 69+10	76
25	9353	69+8, 77+4, 85+2, 91−4	83
July 2	9360	85+1	86
12	9370	91+4, 96−3	94
27	9385	*98+7, *91−3	?96

Date.	J.D. 240 +.	Corrected Comparisons.	Deduced Mag.
1884			
Aug. 1	9390	103+0	103
6	9395	103+0 or fainter	?103
11	9400	108+0	108
14	9403	108+0	108
19	9408	108+4	112
23	9412	108+4	112
25	9414	108+4	112
Sept. 1	9421	108+2	110
5	9425	About 108+0	?108
13	9433	98+2	100
18	9438	98+1	99
20	9440	96+5, 98−1	99
Oct. 5	9455	78+1, 77−1	78
12	9462	69+6, 78−7	73
Nov. 7	9488	64−4	60
10	9491	64−4	60
13	9494	64−2	62
17	9498	64−2	62
19	9500	64−2	62
28	9509	64+4	68
Dec. 8	9519	77+7	84
20	9531	85+4, 91−3	88
1885			
Apr. 8	9640	68+6, 78−4, 77−2	74
17	9649	69+5, 68+1	72
19	9651	68+3, 77−5, 69+3	72
23	9655	69+1	70
26	9658	69+3, 77−4	72
May 7	9669	69+5	74
10	9672	69+3	72
16	9678	77−2	75
June 1	9694	91−2, 98−6	90
9	9702	96+1, 98−1	97
July 4	9727	108+4	112
25	9748	Invisible <108+2	

* ? Designations accidentally transposed.

TABLE III.—*Baxendell's Observations of T Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1885			
July 29	9752	108+5 (only glimpsed)	113
Aug. 3	9757	108+5, min. vis. −5	113
14	9768	108+0	108
20	9774	103+1, 108−4	104
28	9782	96+5, 98+0	100
Sept. 3	9788	96+3, 98−3	97
13	9798	85+1, 91−3	87
25	9810	64+9, 69+1	72
27	9812	69+0	69
Oct. 29	9844	64+6, 69+1	70
Nov. 8	9854	77+4, 85−3	82
15	9861	85−2	83
19	9865	85+2, 91−5	86
Dec. 1	9877	96+1, 98−1	97
26	9902	108+3	111

Date.	J.D. 241+.	Corrected Comparisons.	Deduced Mag.
1886			
	241+		
Apr. 6	0003	69+2	71
26	0023	77−1	76
May 21	0048	96+2, 103−1	100
Sept. 14	0164	69−3	66
Oct. 23	0203	96−3	93
Nov. 2	0213	96+8, 103+0	104
16	0227	108+3	111
30	0241	108+6	114
Dec. 4	0245	108+6	114
25	0266	108+0	108
29	0270	108−4	104
1887			
Jan. 12	0284	91−3	88

The observations have been arranged and tabulated in the same manner as in the case of the stars dealt with in previous instalments of this series, the length of period assumed being 165 days. Table IV. shows the means for the twelve portions for different groups of periods, in which all the observations are included except the first ten, those belonging to periods I. and IX.

These means show an apparent advance of epoch of maximum between "A" and "B" of about one portion, or nearly 14 days in 7 periods; while between "B" and "C" the epoch has apparently *receded* two portions, or $27\frac{1}{2}$ days in 25 periods. The position of "D" and "E" is identical with that of "C," so that there is no obvious sign of any variation of period during these 20 periods. The subscript figures give the number of periods in which the observations occur.

A mean curve has been formed by shifting "B" one portion back, and "C," "D," and "E" one portion forward; the resulting means being given at the foot of Table IV.

TABLE IV.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12
A (X. to XVIII.)	107_4	98_1	$83{\cdot}5_4$	70_6	66_7	66_5	73_6	83_3	89_1	108_1	108_1	113_1
B (XIX. to XXIII.)	119_1	108_3	96_2	83_1	74_1	$68{\cdot}5_2$	70_2	76_2	$94{\cdot}5_2$	98_1	$107{\cdot}5_2$	111_3
C (XLIII. to XLIX.)	95_5	84_4	74_5	67_5	69_5	73_2	87_4	$96{\cdot}5_2$	$103{\cdot}5_4$	111_3	112_5	103_4
D (L. to LVI.)	97_3	81_2	70_3	67_2	69_3	79_5	$90{\cdot}5_4$	102_3	106_3	112_3	113_5	101_2
E (LVII. to LXII.)	102_3	87_3	$76{\cdot}5_2$	67_3	68_4	74_3	$84{\cdot}5_4$	96_3	105_3	111_2	$113{\cdot}5_2$	110_2
Mean, with shifted positions.	106_{15}	97_{14}	84_{14}	72_{17}	67_{19}	68_{19}	75_{18}	87_{17}	97_{10}	105_{13}	111_{12}	113_{14}

Each observation has then been compared with the curve thus formed, and Table V. gives the mean result for each group of each portion. As the differences would have been too large to be trustworthy, had the observations been retained in their original position, they have been shifted in the same manner as that which was adopted in forming the mean curve, before comparing them with the latter. There is thus a general correction of −14 days to be applied to the periods contained in "B," and one of +14 days to those in "C," "D," and "E."

The place of maximum, as read from the mean curve, is 68 days after the point which corresponds with the initial date of each period.

TABLE V.

Comparison of the Observations with a Mean Light Curve.

(The unit is 0·1 magnitude.)

Period.	Initial Date 240+.	Reference Numbers and Mean Magnitudes of Portions. (1). 107.	(2.) 97.	(3.) 84.	(4.) 72.	(5.) 67.	(6.) 68.	(7.) 75.	(8.) 87.	(9.) 97.	(10.) 105.	(11.) 111.	(12. 113
I.	0214	...	...	...	$+12_3$	$[+16_3]$	$+20_1$	$+18_1$	$+11_1$	...	...	...	...
IX.	1520	...	...	...	...	...	...	...	...	...	$+20_1$	...	...
X.	1685	...	...	...	-6_2	-6_1	$[-5_1]$	-2_1	$-2{\cdot}5_1$	...	$+5_1$	...	0_1
XII.	2015	$+3{\cdot}5_1$	...	...	-2_2	$[-3_2]$	...	-1_2	...	...	...	...	...
XIV.	2345	...	...	$+8_1$	$+1_2$	$[0]_3$	$[0]_2$	-1_2	...	...	...	...	...
XV.	2510	$+4_1$	...	-1_1	-2_2	$+0{\cdot}5_1$	...	...	...	...	...	...	...
XVI.	2675	...	...	-6_1	-2_3	$[+1_3]$	$[+0{\cdot}5_2]$	-3_1	-4_1	...	...	-1_1	...
XVII.	2840	$+2_3$	0_2	-5_3	-5_4	$[-3{\cdot}5_1]$	$[-3_3]$	-7_3	...	-4_1	...	...	...
XVIII.	3005	...	...	...	...	$[+2_1]$	$[+2]_1$	$+2_1$	-2_2	...	...	...	$[+6]$
XIX.	3184	$[+7{\cdot}5_1]$	-1_1	-5_3	-1_2	$[+1_1]$	0_1	$+6_1$	$+4_1$	..	...	...	...
XX.	3349	...	...	...	...	...	...	0_1	...	$+2_1$	$+2_1$	$+3_3$	...
XXI.	3514	$-2{\cdot}5_3$	-5_1	...	...	$[+1_2]$	$[+3]_1$	...	0_1	...	$+1_1$	-4_1	...
XXII.	3679	...	...	...	...	...	...	...	...	...	...	$+5_1$	...
XXIII.	3844	0_1	...	...	...	...	...	...	...	...	...	..	...
XLIII.	7116	...	$+6_1$	$+8_1$	$+9{\cdot}5_2$	$+4_1$	$(+3)_3$	0_2	$-2{\cdot}5_4$	-4_2	-3_3	...	[+1
XLIV.	7281	...	-3_1	-4_1	-2_1	...	...	...	...	...	...	...	[+1]
XLV.	7446	...	...	...	...	...	...	...	...	...	-1_3	$[+2_3]$	[0
XLVI.	7611	0_4	-1_2	...	$+6_2$	$(+1)_3$	$(0)_3$	...	...	...	...	...	...
XLVII.	7776	...	-10_1	-4_2	$+3_1$	$[+1_2]$	$[0]_4$	-3_3	-5_1	...	-1_1	-1_2	[−5
XLVIII.	7941	-11_1	-10_2	-8_1	-10_3	(-5_3)	$-0{\cdot}5_2$	...	-2_3	...	...	...	...
XLIX.	8106	...	...	...	...	$[-3]_2$	-2_1	...	$+5_2$	$+3_2$	$+8_1$	$+2_1$	[0
L.	8271	-4_1	-3_2	...	$+1_2$	$[0]_1$	$[0]_1$	$+2_1$	$+7_2$	$+6_1$	$+7_1$	$+3_1$	-1
LI.	8436	...	...	...	...	...	...	$+4_1$	$+3_2$	$+1_1$	...	$+1_3$	[+5
LII.	8601	...	...	$+1_1$	-2_1	...	$[+3]_1$	$+2_1$	$+3{\cdot}5_1$	...	...	...	..

TABLE V.—*Comparison of the Observations with a Mean Light Curve*—continued.

(The unit is 0·1 magnitude.)

Period.	Initial Date 240+.	Reference Numbers and Mean Magnitudes of Portions. (1.) 107.	(2.) 97.	(3.) 84.	(4.) 72.	(5.) 67.	(6.) 68.	(7.) 75.	(8.) 87.	(9.) 97.	(10.) 105.	(11.) 111.	(12.) 113.
LIII.	8766	...	...	...	...	...	..	...	...	...	$+3_1$	...	...
LIV.	8931	...	...	...	...	...	...	$+8_1$	...	$+8_1$	...	...	$[+1]_1$
LV.	9096	...	-1_1	...	...	...	...	...	...	...	...	...	$(-1)_2$
LVI.	9261	0_1	-3_1	-4_1	-1_1	$[-1]_4$	$(+2)_2$	$+5_2$	$+3_2$	...	-3_1	-1_4	$[-2]_3$
LVII.	9426	-6_1	-3_1	-12_2	...	$[-6_3]$	$[-5_2]$	$+2_2$	0_1	...	...	...	...
LVIII.	9591	...	...	...	$+2_1$	$[+4_4]$	$+3_2$	$+2_1$	$+4_1$	$+4_1$	$+4_1$	...	$[0]_1$
LIX.	9756	$+3_2$	$+5_2$	$+10_1$	$+6_2$	$[+1]_1$	...	-4_1	$-3·5_3$	-3_1	...	0_1	...
LX.	9921	...	...	...	...	...	0_1	...	-9_1	...	-3_1	...	...
	241+												
LXI.	0086	...	...	...	...	...	-3_1	...	...	-4_1	0_1	$+1_1$	$[+1]_2$
LXII.	0251	...	$+6_2$	$+2_1$	...	...	...	...	...	...	...	...	...
$p-f$		+16	+23	+25	+17	(+4) +10	(−8) −14	−19	−22	−18	−14	−8	(+4) +12

In obtaining corrections to epoch in the usual manner, the observations in square brackets have not been used, and some only of those in round brackets, the unused ones being too near max. or min. to be of any assistance; and the lower figures for $p-f$ have been taken in portions 5, 6, and 12, as being more correct for the parts of those portions which alone were used. It is very unlucky that the first 6 observations (which would have been so specially valuable) are hard to understand. There are three hypotheses. Either (1) the identity of the star was mistaken, in which case they mean nothing; or (2) the star was at its maximum about 240 0278, but was nearly 1·2 magnitudes fainter

TABLE VI.

Corrections to Assumed Epoch.

Period.	Assumed Epoch.	Correction. d.	Period.	Assumed Epoch.	Correction. d.
I.	0268·0	+ 9·0	XLIII.	7198·0	− 7·8
X.	1753·0	− 2·7	XLVII.	7858·0	−14·9
XII.	2083·0	− 0·6	XLVIII.	8023·0	−18·6
XIV.	2413·0	+ 4·9	L.	8353·0	−19·5
XVI.	2743·0	− 0·6	LII.	8683·0	−15·9
XVII.	2908·0	+ 0·3	LVI.	9343·0	−17·0
XIX.	3238·0	+10·1	LVII.	9508·0	−19·6
XXI.	3568·0	+11·7	LVIII.	9673·0	−14·7
			LIX.	9838·0	− 7·5
			LX. to LXII.	0168·0	− 8·7

than usual; or (3) it remained nearly stationary for three weeks at a considerable (but unknown) interval after maximum, a proceeding of which there is no other instance. The observations have been treated on the basis of (2), but the result must be regarded as extremely doubtful. The *general* difference in brightness between one period and another has been taken account of in working out these corrections to epoch.

By combining different pairs we get an idea of the changes of period, and the following formula was found by trial and error:

$$240\ 0116{\cdot}4 + 164{\cdot}6E + 13 \sin (8^\circ{\cdot}5E + 211^\circ).$$

An alternative value of $A = 4^\circ$ instead of $A = 8^\circ{\cdot}5$ was tried with some success, but did not suit all the observations.

Table VII. shows the comparison with this formula and with that of Chandler, which is

$$240\ 3399 + 165{\cdot}0E + 10 \sin (5^\circ E + 110^\circ).$$

For a period of 165 days the correlation formula (*Mon. Not.*, lxviii. pp. 551–2) would be $18 \sin (5^\circ{\cdot}6E + \text{const.})$.

TABLE VII.

Comparison of Epochs of Maxima.

Period.	Baxendell 240+.	Calculated.	B–C.	Chandler.	Bax.–Ch.
I.	(0277·0)	0272·7	(+4·3)	0266·6	(+10·4)
X.	1750·3	1750·7	−0·4	1757·7	− 7·4
XII.	2082·4	2082·1	+0·3	2088·4	− 6·0
XIV.	2417·9	2414·3	+3·6	2418·8	− 0·9
XVI.	2742·4	2747·1	−4·7	2749·0	− 6·6
XVII.	2908·3	2913·6	−5·3	2914·0	+ 5·7
XIX.	3248·1	3246·6	+1·5	3243·7	+ 4·4
XXI.	3579·7	3579·3	+0·4	3573·1	+ 6·6
XLIII.	7190·2	7186·5	+3·7	7186·9	+ 3·3
XLVII.	7843·1	7840·3	+2·8	7844·9	− 1·8
XLVIII.	8004·4	8004·4	0·0	8009·6	− 5·2
L.	8333·5	8333·5	0·0	8339·2	− 5·7
LII.	8667·1	8663·6	+3·5	8669·0	− 1·9
LVI.	9326·0	9326·9	−0·9	9329·6	− 3·6
LVII.	9488·4	9493·2	−4·8	9494·9	− 6·5
LVIII.	9658·3	9659·6	−1·3	9660·3	− 2·0
LIX.	9830·5	9826·1	+4·4	9825·8	+ 4·7
LXI.	0159·3	0159·1	+0·2	0156·9	+ 2·4

The differences between Baxendell and Chandler, though never great, are clearly systematic; but the new formula was found

quite independently, before Chandler's results had been worked out. It will, of course, require confirmation or modification from comparison with the results of other observations. It is perhaps worth noting that in those cases where "B–C" consists of several days, an inspection of the observations shows that the true maximum might well be in accordance with the formula. Nevertheless, it is of course possible that the true curve of variation of epoch will eventually prove not to be a simple sine curve, as the Rev. T. E. R. Phillips has already suggested in another case.

The analysis of the mean curves gives

$$23 \cos(\theta - 313^\circ) + 1{\cdot}62 \cos 2(\theta - 16^\circ) + 0{\cdot}37 \cos 3(\theta - 99^\circ),$$

which, when divided by the range and multiplied by 6, becomes

$$2{\cdot}94 \cos(\theta - 313^\circ) + 0{\cdot}21 \cos 2(\theta - 16^\circ) + 0{\cdot}05 \cos 3(\theta - 99^\circ).$$

This expression would bring the epoch of maximum to one day earlier than it was read from the curve. If this be correct, the initial date of the formula would be about 240 0115·4 instead of 240 0116·4, and all the epochs in B and C would be correspondingly changed.

The results of each group of periods have been analysed, except group B, in which the number of observations is so small that it would not be worth while to do it. The following figures have been obtained, referred to the arbitrary zero point :—

Mean Curve : $2{\cdot}94 \cos(\theta - 313^\circ) + 0{\cdot}21 \cos 2(\theta - 16^\circ) + 0{\cdot}05 \cos 3(\theta - 99^\circ)$

Group A : $2{\cdot}97 \cos(\theta - 318^\circ) + 0{\cdot}39 \cos 2(\theta - 1^\circ) + 0{\cdot}135 \cos 3(\theta - 37^\circ)$

,, C : $2{\cdot}86 \cos(\theta - 283^\circ) + 0{\cdot}07 \cos 2(\theta - 8^\circ) + 0{\cdot}124 \cos 3(\theta - 55^\circ)$

,, D : $2{\cdot}90 \cos(\theta - 275^\circ) + 0{\cdot}24 \cos 2(\theta - 173^\circ) + 0{\cdot}04 \cos 3(\theta - 42^\circ)$

,, E : $2{\cdot}91 \cos(\theta - 291^\circ) + 0{\cdot}22 \cos 2(\theta - 177^\circ) + 0{\cdot}023 \cos 3(\theta - 99^\circ)$

The differences in phase in the principal term are in very close agreement with the corrections to epoch.

No. 14, *R Leonis* (3493 *Chandler*, 094211 *Harvard*).

$9^h\ 39^m\ 45^s\ +12^\circ\ 5'{\cdot}9$ (1855·0).

$9\ 42\ 11\ +11\ 53\ '6$ (1900·0).

Identification of Comparison Stars.

In this case Baxendell kept the same designations for the comparison stars throughout, except in the case of the star A.S.V. 6, which was sometimes called "star to the south," sometimes "star ½ degree south," sometimes "star above half a degree distant," sometimes "7·9 mag. star," and sometimes "β." In 1866 April 13 to May 15, the stars "a" and "b" are called "old a and b," and a note is added to the "old a" of April 13, now "b?" But the deduced magnitudes given by Baxendell show that the same a and b are used throughout. There are two

diagrams, and eight rough sketches, which all agree in the designations of the comparison stars, except that in one case Mayer's 419 is called 417 by an obvious slip of the pen. The same slip seems to have occurred in the observation of 1880 Dec. 1.

TABLE VIII.

Comparison Stars.

Bax.	A.S.V.		Harvard.		Adopted Mag.
	No.	Grade.	Letter.	Mag.	
417	1	...	...	5·84	58
419	2	...	...	6·43	64
β	6	...	...	7·47	75
a	16	...	...	8·98	90
b	21	...	...	9·64	96
c	...	...	...	...	115

The magnitudes adopted are those of Harvard for the first five stars; that of *c* is calculated from the "limiting aperture" given

TABLE IX.

Baxendell's Differences between his Comparison Stars.

(Subscript figures give numbers of observations; the unit is approximately 0·1 mag.)

	64–58.	75–64.	90–75.	90–64.	96–90.	115–96.
1840–2	$5·7_3$	...	...	...	$8·0_1$	...
1848–9	$6·4_{17}$	...	...	$21·4_5$	$7·0_2$	$20·0_1$
1850–3	$6·2_7$	...	...	$23·4_{10}$	$5·6_{19}$	$16·4_5$
1854–5	$8·2_5$	...	...	$24·8_{13}$	$6·8_9$	$18·0_1$
1857–9	$5·1_{18}$	...	...	$22·1_{14}$	$2·9_9$	$19·5_4$
1860–3	$5·2_6$	$11·6_8$	$12·8_7$	$23·4_9$	$4·2_{14}$	$17·0_3$
1864–6	$4·7_{23}$	$10·3_{14}$	$12·5_2$	$19·0_1$	$6·1_4$	...
1868–71	$5·0_1$	...	...	$25·7_3$	$6·0_3$	...
1878–82	$3·5_{17}$	...	...	$23·4_{10}$	$6·5_8$	...
1883–7	$4·1_9$	$14·5_1$	...	$24·5_5$	$6·4_6$	$29·3_5$
Mean of first series (old notation)	$6·6_{32}$	...	...	$23·7_{28}$	$6·1_{31}$	$17·2_7$
Second series	$4·9_{38}$	$10·8_{22}$	$12·7_9$	$22·8_{27}$	$4·2_{30}$	$18·4_7$
Third series	$3·7_{26}$	$14·5_1$	...	$23·8_{15}$	$6·5_{14}$	$29·3_5$
General Mean	$5·1_{96}$	$11·0_{23}$	$12·7_9$	$23·4_{70}$	$5·4_{75}$	$20·8_{19}$
Harvard interval (unit, tenths of mag.).	5·9	10·4	15·1	25·5	6·6	...

for all the above stars except β on 1858 Jan. 16. From Table IX. it will be seen that Baxendell did not, in this case, make his steps larger than tenths of a magnitude, but if anything smaller. The usual scale of reduction has therefore not been applied, except in the year 1887, when his steps appear to be of the same character as in his observations of other stars.

Up to the end of 1856, Baxendell used a system of symbols in his comparisons, instead of figures. These symbols are the same as those used by W. Herschel; but on investigation it soon became apparent (1) that they do not represent the same value in magnitudes as with him; (2) that their value in magnitudes changes with the brightness of the stars compared *or* with the instrument and power used—it is difficult to say which. As he made a very large number of comparisons between bright stars with the naked eye and with opera glasses, it has proved possible to obtain a very satisfactory table of values for his symbols for stars of the higher magnitudes. (The magnitudes used in the investigation were the Oxford ones.)

TABLE X.

Corresponding Value in Magnitude.

Symbol.	A. For Mags. above 3·8.	B. For Mags. from 3·8 to 6·5.	C.	D.
.	·05 (14)	·10 (60)	·2	·3
;	·09 (37)	·15 (111)	·3	·4
,	·13 (98)	·20 (145)	·4	·5
_, or ¯,	·18 (22)	·25 (55)	·5	·6
—	·23 (68)	·30 (152)	·6	·7
=, or =,	·27 (17)	·40 (40)	·7	·8
—,	·36 (22)	·50 (119)	·8	1·0
=, _	(not used)	·60 (9)	1·0	1·2
¯ —	·56 (3)	·70 (5)	(not used)	(not used)
— —	·60 (4)	·80 (34)	1·5	1·8
— — —	·90	(not used)	1·8	2·1

The figures in brackets give numbers of comparisons. The values obtained by calculation have been slightly "smoothed."

For magnitudes below 6·5 there is a large and apparently sudden increase in the value of the symbols, which may be due to the observations being made always through a telescope, and with powers of not less than 30. After about magnitude 9·5 a further increase appears to take place. But the observations available for magnitudes below 6·5 are comparatively very scanty, and a much greater degree of uncertainty must attach to the values found for them. [In this connection it may be well to explain that the earlier observations of R Coronæ (in the second instalment of this series) were recorded in this notation, and as they were discovered only just in time to add them to the record for print-

ing, the symbols were interpreted (in haste) by the table (B) given above for magnitudes between 3·8 and 6·5. As the magnitude was not much above 6·5 even at maximum, and below it at other times, the intervals between the variable and the comparison stars should probably be a little greater than they are given. The only important result would be to make the variable come out about a tenth of a magnitude (or rather more) brighter near its maximum. A few additional early observations of this variable have also been found.]

In dealing with the observations of R Leonis, the symbols have been interpreted by (Table X.) B when the magnitude is brighter than 6·5; by C when it lies between 6·5 and 9·6; and by D when it is less bright than 9·6. An inspection of Table IX. and of the means of groups A to C (p. 171) will show to what extent the result may be considered satisfactory. As the interval between the two brightest stars comes out greater, and the intervals between the fainter ones less, than the Harvard intervals, it does not seem likely that the changes in the value of Baxendell's symbols (shown in B, C, and D) have been at all exaggerated. The mean interval for 115–96 in the third series of observations is inflated by what appears to be probably an error in the observations of 1886 May 14. Occasionally, during the early period, Baxendell used a proportional method of comparison, either in the form *a* (*m*) R (*n*) *b*, or *a* @ R = R @ *b* (sometimes *a* @ R = or > R @ *b*), and so on.

Baxendell used a greater variety of instruments and powers than usual. Until 1857 June 15, 30 (*a*) was generally used. Mr. Worthington's $4\frac{1}{4}$-in. achromatic, with power 40, was used on 1849 May 11. On 1846 Jan. 4–20, and 1849 Mar. 16 and Apr. 30, a $4\frac{1}{2}$-in. reflector was used; with power 37 on the two last dates. A 6-inch reflector was used 1849 Dec. 22; 1850 Jan. 14, Feb. 3, and Mar. 4 and 29; 1851 Feb. 1, Mar. 29, Dec. 22; 1852 Jan. 17 and 25, Feb. 9, 21, 22, 24, 28, Mar. 2, 7 and 8, and Dec. 18 and 27; 1855 Jan. 14 (2nd observation), 28 and 29; throughout March, April and May, and Dec. 29, and 1856 Jan. (powers 37 on the first three dates, and 1855 Jan. 28, Mar. 6, Apr. 21 and May 8; 130 on the rest, except 1851 Feb. 1, 40). The finder was used 1848 Dec. 13.

A 68 was used 1857 May 31, June 13, 14, 17 and 18; 1858 Mar. 11, Apr. 20, 26, June 1, 5, and 7; 1860 Apr. 15 and 23; 1861 Jan. 15, Mar. 30, Apr. 3, 28, May 17, 27; 1862, from Apr. 10; 1863, from Mar. 15 to May 22 (except Apr. 30); 1864, from Feb. 5; and from 1865 Jan. 20 to 1869 Apr. 30. A 40: 1858 Mar. 16, 20, 25, Apr. 12. A 35: 1858 Jan. 20. C 39 was used 1861 Jan. 1, Feb. 6 to 28, Mar. 8, Apr. 9 and 16, and Dec. 25; 1863 Feb. 14 and 22, Apr. 30, Nov. 5 to Dec. 17; 1864 Jan. 2 and 28; 1869 Dec. 25. T 35 was used 1857, from Nov. 18; 1858, to Jan 16, Feb. 6, 7 (2nd observation) and 17 to Mar. 7: and T 40, 1858 Jan. 25. R (13) 81 was used 1858 Feb. 7, (1st observation), Mar 23. Apr. 1, 6, 10, 17, May 1 to 18; 1860

Mar. 14, May 3; 1861 Mar. 13, Apr. 17, May 9; 1862 Mar. 3; 1863 Feb. 8: and R (13) 196 on 1858 Apr. 13, 21 and 23. R (12) was used 1858 Feb. 17, Mar. 21: and 22 (*a*) was used 1858 Dec. 2, 18; 1859 Jan. 6, 23, and Feb. 7 to May 5 (power 21 on Jan. 6 and Feb 23), May 19, Dec. 21 (39); 1860 Jan. 6 and 15, Feb. 9 to 27, and Apr. 8 (power 39 on Feb. 9 and 16, and Apr. 8).

In 1870 and 1871, Mr. Gladstone's 7¼-in. achromatic was used; and from 1878 onwards, E 80. On 1878 Mar. 13 and 27 it is noted that "Dancer eyepiece" was used. On several occasions an observation was made also with the finder, the result being usually very different from that obtained with the larger instruments. These have not been given in Table III. nor used in the discussion.

TABLE XI.

Baxendell's Observations of R Leonis.

(At first called by him "Mayer's 420.")

Date.	J.D. 239+.	Comparisons.	Deduced Mag.
1837			
Jan. 10	2020	Glimpsed.	
1840			
Feb. 10	3146	<64	
14	3150	<64	
Apr. 16	3212	64+2	66
30	3226	64−3	61
May 2	3228	58+4, 64−3	62
1842			
May 2	3958	90−8 <64 (much)	82
1846			
Jan. 1	5298	Invis. <96	
4	5301	,, (with A 30).	
4	,,	Visible <96 (with R(4½))	
20	5317	Visible, about 100	
Feb. 26	5354	90+0, >96	90
1848			
Nov. 4	6336	64−1	63
13	6345	64−3, 58+4	62
17	6349	64−5	59
18	6350	58+4, 64−5	60
30	6362	58+3, 64−5	60
1848			
Dec. 2	6364	58+2, 64−5	60
6	6368	58−1, 64−6	58
9	6371	58−1, 64−8	56
13	6375	58−2, 64−8	56
16	6378	58−1	57
21	6383	58+1, 64−5	59
27	6389	58+1, 64−5	59
1849			
Jan. 2	6395	58+1, 64−5	59
5	6398	58+2	60
11	6404	58+3, 64−3	61
14	6407	58+3, 64−2	62
17	6410	58+5, 64−2	62
20	6413	64+2	66
22	6415	64+1	65
30	6423	64+1	65
Feb. 8	6432	64+8	72
12	6436	64+8, 90−15	74
19	6443	64+8, 90−15	74
27	6451	64+15	79
28	6452	64+15, 90−7	81
Mar. 3	6455	90−8	82
16	6468	64+15, 90−5	83

TABLE XI.—*Baxendell's Observations of R Leonis*—continued.

Date.	J.D. 239+.	Comparisons.	Deduced Mag.
1849			
Apr. 1	6484	Invis. <96	
30	6513	About 96+0, <90	96
May 11	6524	96+10, 115−10	106
Oct. 13	6679	58+3, 64−2	62
15	6681	58+3, 64−3	61
21	6687	58+5, 64−3	62
28	6694	64+1	65
29	6695	64+3	67
Nov. 13	6710	64+4	68
Dec. 6	6733	64+15	79
22	6749	64+15, 90−4	82
27	6754	90−3	87
1850			
Jan. 14	6772	90+2, 96−4	92
Feb. 3	6792	90+8, 96+3, 115−18	98
Mar. 4	6821	96+10, 115−4	108
29	6846	96+8, 115−5	107
May 12	6890	<96	
Oct. 11	7042	64+10, 90−14	75
25	7056	64+14, 90−8	80
Nov. 7	7069	64+14, 90−8	80
20	7082	64+16, 90−6	82
27	7089	64+16, 90−5	82
Dec. 12	7104	90+4, 96−4	93
1851			
Jan. 7	7130	Invis. <90	
28	7151	,, <96	
Feb. 1	7155	96+8, 115−10	104
Mar. 1	7183	Glimpsed <96	
29	7211	90+2, 96−4	92
May 28	7271	64+12	76
29	7272	64+6	70
Nov. 21	7448	96+0 (or <96)	96
Dec. 22	7479	96+5, 115−10	103
1852			
Jan. 17	7505	96−5, 90+2	92
1852			
Jan. 25	7513	90−4	86
Feb. 9	7528	90−6	84
15	7534	<90	
21	7540	90−5	85
22	7541	90−4	86
24	7543	90−6	84
28	7547	64+18, 90−7	82
Mar. 2	7550	90−8	82
7	7555	90−8	82
8	7556	90−8	82
28	7576	64+4, 90−18	70
Apr. 3	7582	64−2	62
13	7592	(58+64)/2	61
15	7594	58+3, 64−2	62
19	7598	(58+64)/2, or less bright	?61
26	7605	58+3, 64−5	60
May 19	7628	(58+64)/2, or brighter	?61
1853			
Jan. 9	7863	90−3	87
Feb. 16	7901	58+5, 64−1	63
16	,,	58+6, 64−2	63
May 2	7976	64+8, 90−15	74
12	7986	64+8, 90−15	74
Dec. 10	8198	64+8, 90−15	74
1854			
Feb. 18	8268	58+5, 64−2	62
25	8275	58+8, 64−2	64
Mar. 1	8279	58+8, 64−1	63
2	8280	64+1	65
4	8282	64+3	67
16	8294	64+8, 90−18	72
21	8299	64+8, 90−15	74
Apr. 2	8311	64+15, 90−15	77
24	8333	64+20, 90−8	83
Dec. 18	8571	64−3	61
27	8580	64−3	61

TABLE XI.—*Baxendell's Observations of R Leonis*—continued.

Date.	J.D. 239+.	Comparisons.	Deduced Mag.
1855			
Jan. 14	8598	64+8, 90−18	72
14	,,	64+6, 90−18	71
28	8612	64+6, 90−18	71
29	8613	64+6, 90−18	71
Feb. 9	8624	64+15, 90−8	80
13	8628	64+15, 90−8	80
Mar. 6	8649	64+20, 90−8	83
6	,,	90−8	82
21	8664	90−4	86
25	8668	90−4	86
Apr. 21	8695	90−3	87
May 8	8712	90+6	96
Nov. 3	8891	58+4, 64−2	62
10	8898	64−2	62
16	8904	64−1	63
27	8915	64+14	78
Dec. 5	8923	64+8, 90−15	74
12	8930	64+15, 90−8	80
21	8939	Barely visible; moonlight and low >90	
29	8947	90−12	78
1856			
Jan. 7	8956	90−6	84
11	8960	90−6	84
28	8977	90−3	87
Mar. 24	9033	Invisible.	
May 9	9079	,, <96	
29	9099	,, (twilight).	
Nov. 29	9283	90+0, or brighter	90 or 89
30	9284	90−4	86
1857			
Jan. 25	9340	Invisible <96	
Mar. 21	9395	?90+3	?93
Apr. 13	9418	64+20, 90−6	84
15	9420	90−5	85
16	9421	90−4	86
19	9424	90−7	83
25	9430	90−8	82
26	9431	64+12, 90−11	78
May 7	9442	64+10	74
15	9450	64+10, 90−15	74
21	9456	64+5	69
24	9459	64+2	66
26	9461	64+3	67
28	9463	64+2 (uncertain)	?66
31	9466	58+2, 64−3	60
June 7	9473	58−2	56
7	,,	58−1	57
13	9479	58−2, 64−8	56
14	9480	58−3, 64−8	56
15	9481	Invisible, twilight.	
17	9483	58−3, 64−9	55
18	9484	58−3, 64−9	55
Nov. 18	9637	96+8	104
23	9642	Just visible.	
26	9645	,, ,, <90	
Dec. 19	9668	120 −5*	115
20	9669	120 −4*	116
23	9672	120 −5*	115
1858			
Jan. 1	9681	Not seen (moonlight).	
9	9689	96+13, 115−7	108
11	9691	90+17, 96+12	†108
16	9696	90+14, 96+9, 115−11, 58+47, 64+39	†104
20	9700	96+10, 115−10	106
25	9705	90+12, 96+7	102

* These comparisons are with the "min. vis.," with Tulley 35, aperture 3·15.

† The observations of January 11 and 16 give the "limiting aperture" for the different stars, from which these comparisons have been calculated.

TABLE XI.—*Baxendell's Observations of R Leonis*—continued.

Date.	J.D. 239+.	Comparisons.		Deduced Mag.
1858				
Feb. 6	9717	96+3,	115−15	100
7	9718	96+0		96
7	,,	96+2		98
14	9725	96+0		96
17	9728	96+0,	90+3	94
22	9733	96+0,	90+3	94
24	9735	90+0		90
28	9739	90−1		89
Mar. 3	9742	90+0		90
7	9746	90+0		90
11	9750	90−1		89
13	9752	90−2		88
16	9755	90−3		87
20	9759	90−4,	64+14	82
21	9760	90−4		86
23	9762	90−7,	64+7	77
25	9764	64+5,	90−15	72
26	9765	64+5,	90−16	72
Apr. 1	9771	64+3		67
4	9774	64+2		66
6	9776	64+2		66
9	9779	64+0		64
10	9780	64+1		65
12	9782	64+0		64
13	9783	64+0		64
17	9787	64+0		64
20	9790	64−1		63
21	9791	64−1		63
23	9793	64−1		63
26	9796	64−2		62
May 1	9801	58+2,	64−2	61
3	9803	64−2		62
4	9804	58+3,	64−2	62
5	9805	(58+64)/2		61
6	9806	64−1		63
10	9810	64−1		63
18	9818	64−2		62
June 1	9832	64+2		66
5	9836	64+2		66
7	9838	64+3		67
	240+			
Dec. 2	0016	96+1		97
18	0032	88−2		86
1859				
Jan. 6	0051	64+17,	90−5	83
18	0063	64+8,	90−15	74
23	0068	64+20,	90−7	84
Feb. 7	0083	64+0		64
11	0087	64+0		64
14	0090	58+2,	64−1	62
23	0099	58+0,	64−3	60
27	0103	58+0,	64−4	59
28	0104	58+0		58
Mar. 7	0111	58+0		58
13	0117	58−2		56
15	0119	58−1		57
18	0122	58−1		57
29	0133	58+3,	64−2	62
31	0135	58+2,	64−2	61
Apr. 3	0138	58+3,	64−2	62
9	0144	58+4,	64−2	62
16	0151	64−1		63
17	0152	64+0		64
18	0153	64+0		64
May 1	0166	64+0		64
5	0170	64+6		70
12	0177	64+3		67
19	0184	64+9		73
20	0185	64+7		71
22	0187	64+6		70
29	0194	64+15,	90−13	78
Dec. 21	0400	64+0		64
1860				
Jan. 6	0416	58+2,	64−3	60
15	0425	58+3,	64−3	61

TABLE XI.—*Baxendell's Observations of R Leonis*—continued.

Date.	J.D. 240+.	Comparisons.	Deduced Mag.
1860			
Jan. 25	0435	58+3, 64−3	61
31	0441	58+3, 64−2	62
Feb. 6	0447	58+4, 64−2	62
9	0450	58+4, 64−2	62
12	0453	64+1	65
16	0457	64+2	66
20	0461	64+2	66
27	0468	64+7	71
Mar. 4	0474	64+4	68
14	0484	64+6	70
18	0488	64+8	72
25	0495	64+8, 90−12	75
Apr. 8	0509	90−5	85
15	0516	90−4	86
23	0524	90−2	88
May 3	0534	90+0	90
1861			
Jan. 1	0777	64+3	67
15	0791	64+4	68
29	0805	64+10	74
Feb. 6	0813	64+9	73
12	0819	64+17, 90−15	78
28	0835	64+20, 90−7	84
Mar. 8	0843	90−7	83
13	0848	90−6	84
30	0865	90−2	88
Apr. 3	0869	90+0	90
9	0875	96+0	96
16	0882	96−1	95
17	0883	90+1, 96−2	92
28	0894	96+2	98
May 9	0905	96+3, 115−8	103
17	0913	96+5	101
27	0923	96+7	103
Dec. 25	1135	64+20, 90−7	84
1862			
Feb. 20	1192	96+2	98
Mar. 3	1203	96+1	97
Apr. 10	1241	96−1	95
29	1260	90+5, 96−2	94
May 16	1277	90−3	87
Dec. 30	1505	90+3, 96−1	94
1863			
Jan. 23	1529	96+0, 116−20	96
Feb. 8	1545	90+0, 96−3	92
14	1551	96+1	97
22	1559	90+4, 96+0	95
Mar. 15	1580	90−2	88
17	1582	90−3	87
21	1586	90−3	87
22	1587	90−3	87
23	1588	90−3	87
26	1591	90−4	86
Apr. 1	1597	90−10, 75+5	80
4	1600	90−13, 75+3	78
11	1607	64+8, 75−2, 90−14	74
13	1609	64+8, 90−12, 75−1	75
15	1611	64+8, 90−14	74
23	1619	64+8, 90−14, 75−2	74
28	1624	64+7, 75−4, 90−16	72
30	1626	64+9, 75−4	72
May 5	1631	64+6, 75−5, 90−17	71
17	1643	64+3, 75−10	66
22	1648	64+2, 75−13	64
Nov. 5	1815	90+3, 96−1	94
8	1818	90+5, 96+0	96
Dec. 2	1842	About 96−1	95
17	1857	96+1	97
1864			
Jan. 2	1873	90−2	88
28	1899	<90	

TABLE XI.—*Baxendell's Observations of R Leonis*—continued.

Date.	J.D. 240+.	Comparisons.		Deduced Mag.
1864				
Feb. 5	1907	64+9,	75−2	73
8	1910	64+9,	75−2	73
12	1914	64+8,	75−3	72
17	1919	64+5,	75−4	70
Mar. 15	1946	58+3,	64−3	61
23	1954	58−1,	64−5	58
28	1959	58−1,	64−5	58
Apr. 11	1973	58−2		56
13	1975	58−2,	64−6	57
18	1980	58−2,	64−6	57
21	1983	58−2,	64−6	57
25	1987	58−2,	64−6	57
28	1990	58−1		57
May 7	1999	58−1,	64−6	58
16	2008	58−1		57
26	2018	58+0,	64−4	59
29	2021	58+1,	64−4	60
Dec. 31	2237	64+8,	90−11,	76
		75+2		
1865				
Jan. 20	2257	64+8,	75−5	71
Feb. 8	2276	58+3,	64−2	62
14	2282	58+2,	64−3	60
Mar. 1	2297	58+0,	64−4	59
9	2305	58+0,	64−5	58
23	2319	58+2,	64−3	60
30	2326	58+3,	64−3	61
Apr. 10	2337	58+4,	64−1	62
15	2342	64+0		64
24	2351	64+2		66
26	2353	64+2		66
May 1	2358	64+3,	75−7	68
15	2372	64+9,	75−3	72
Dec. 14	2585	64+10,	75−2	74
26	2597	64+1,	75−10	65
1866				
Jan. 15	2617	58+3,	64−2	62
Jan. 16	2618	58+2,	64−3	60
23	2625	58+3,	64−3	61
Feb. 2	2635	58+1,	64−3	60
13	2646	58+2,	64−2	61
17	2650	58+4,	64+0	63
20	2653	64+0		64
25	2658	64+2,	75−8	66
Mar. 13	2674	64+5,	75−3	70
18	2679	64+8,	75+2	74
25	2686	64+14,	75+1	77
31	2692	75+4,	90−8	80
Apr. 13	2705	90−3		87
18	2710	90−2		88
21	2713	90−2		88
29	2721	90−1		89
May 15	2737	96+1		97
18	2740	96+0		96
23	2745	90+5,	96−1	95
Dec. 7	2943	58+0,	64−4	59
1867				
Jan. 28	2995	64+7		71
Mar. 16	3042	90−1		89
1868				
Feb. 3	3366	96+6		102
15	3378	96+5		101
Mar. 5	3397	<96+10	Not seen.	
15	3407	115−7		108
1869				
Feb. 5	3734	96+10		106
Mar. 2	3759	? 96+0*		? 96
6	3763	90+2,	96−4	92
Apr. 26	3814	64+15,	90−12	78
28	3816	64+15,	90−10	80
30	3818	64+17,	90−8	82
Dec. 25	4057	96+12		108

* The entry is "$a=b$," an evident error, which has been conjecturally amended as "$R=b$."

TABLE XI.—*Baxendell's Observations of R Leonis*—continued.

Date.	J.D. 240+.	Comparisons.		Deduced Mag.
1870				
Mar. 25	4147	?64+4*		?68
1871				
Mar. 3	4490	58+2,	64−3	60
Apr. 8	4526	64−2		62
1878				
Jan. 10	6995	58+0,	64−2	60
25	7010	58+0,	64−3	60
31	7016	58+1,	64−3	60
Feb. 15	7031	64+2		66
18	7034	64+2		66
20	7036	64+2		66
Mar. 13	7057	64+8,	90−15	74
27	7071	64+13,	90−7	80
30	7074	90−5		85
Nov. 4	7293	64+1		65
28	7317	58−2,	64−4	58
Dec. 23	7342	58−1,	64−3	59
1879				
Jan. 19	7369	64+0		64
Mar. 31	7440	90−3		87
Apr. 10	7450	90+3,	96−3	93
Nov. 27	7681	64+2		66
28	7682	64+2		66
Dec. 24	7708	58+8		66
1880				
Feb. 5	7751	90+0		90
10	7756	90+4,	96−2	94
Mar. 8	7783	96−1		95
17	7792	96+0		96
Apr. 1	7807	96+4		100
9	7815	96+6		102
17	7823	96+7		103
24	7830	96+0		96
29	7835	96+1		97
May 1	7837	96+0		96

Date.	J.D. 240+.	Comparisons.		Deduced Mag.
1880				
May 3	7839	96+0		96
6	7842	96+0		96
7	7843	96−1		95
12	7848	90+2,	96−4	92
15	7851	90+1,	96−5	91
27	7863	90−8		82
Nov. 19	8039	64+4		68
27	8047	64+10,	90−7	78
Dec. 1	8051	?58+8†,	90−8	74
				?77
1881				
Jan. 5	8086	90−2		88
Feb. 6	8118	96+8,	?−5	104
23	8135	97+13,	115−7	109
Mar. 28	8168	90+2,	96−5	92
31	8171	90+1,	96−5	91
Apr. 6	8177	90−2		88
17	8188	90−3		87
20	8191	90−7		83
May 4	8205	90−6		84
7	8208	64+10,	90−12	76
12	8213	64+13,	90−10	78
20	8221	64+12,	90−14	76
22	8223	64+12,	90−14	76
29	8230	64+10,	90−20	72
1882				
Feb. 12	8489	90+4,	96−3	94
Mar. 12	8517	90−7		83
18	8523	90−5		85
Apr. 6	8542	64+12,	90−13	76
10	8546	64+4,	90−18	70
20	8556	58+2,	64−6	59
May 4	8570	58−4,	64−10	54
6	8572	58−5,	64−8	54
12	8578	58−3,	64−7	56
14	8580	58−5,	64−7	55

* The entry is "419 4½a," with a note: "Should a be R?"
† It seems probable that this should be "64+8."

TABLE XI.—*Baxendell's Observations of R Leonis*—continued.

Date.	J.D. 240+.	Comparisons.		Deduced Mag.
1882				
May 16	8582	58−4,	64−8	55
19	8585	58−5,	64−7	55
25	8591	58−6,	64−10	53
29	8595	58−7,	64−9	53
1883				
Feb. 22	8864	64+5		69
Mar. 3	8873	58+3,	64−1	62
5	8875	64−3		61
12	8882	64−4,	58+0	59
24	8894	58−2,	64−6	57
26	8896	58−3,	64−7	56
Apr. 1	8902	58−3,	64−7	56
8	8909	58−3,	64−7	56
1884				
Jan. 20	9196	64+0		64
Feb. 2	9209	64−1		63
24	9231	64−1		63
Mar. 3	9239	64+2		66
15	9251	64+15		?79*
16	9252	64+4		68
23	9259	64+7		71
Apr. 7	9274	64+8,	90−20	71
10	9277	64+9		73
12	9279	64+11,	75−4	73
May 4	9301	64+16,	90−16	77
19	9316	90−6		84
26	9323	90−5		85
Dec. 14	9525	58−3,	64−7	56
20	9531	58−1,	64−5	58
1885				
Jan. 5	9547	58−2,	64−6	57
Feb. 3	9576	64+7		71
9	9582	64+2		66
22	9595	64+5		69
Mar. 6	9607	64+5,	90−14	72
Apr. 15	9647	90−1		89
May 11	9673	96+1		97
1886				
Jan. 5	9912	64+5,	90−13	73
29	9936	90−5		85
Feb. 14	9952	90−3		87
Mar. 3	9969	96+2		98
8	9974	96+5		101
11	9977	90+7,	96+1	97
29	9995	96+11		107
	241+			
Apr. 9	0006	96+12		108
26	0023	96+10,	115−4	108
May 14	0041†	96+12,	115−25	?99
Dec. 29	0270	90+4,	96−3	94
1887				
Jan. 26	0298	96+9		105
Feb. 11	0314	96+11,	115−11	106
18	0321	96+14,	115−11	107
26	0329	96+18,	115−6	112

Chandler's mean period being 312·9 days, a period of 312 days was assumed, and the whole series of observations was divided into portions of 26 days, and tabulated in the usual manner, taking the mean of all observations in each portion.

The means of large groups of periods were taken, as follows, in units of 0·1 magnitude.

Group.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
A	92_3	85_2	75_2	64_4	60_2	63_4	73_6	80_5	85_4	93_3	102_3	103_1
B	92_4	81_6	72_5	62_7	58_6	63_6	68_4	77_5	86_2	95_4	97_4	97_4
C	103_4	102_2	91_4	82_2	70_4	60_4	58_5	63_3	69_3	74_4	88_4	93_5

* "Observation doubtful." † This looks like an error.

The subscript figures give the number of periods in which the observations occur. Group A consists of periods – VIII. to 0 (as given in Table XII.), omitting the observations of 1849 Dec. 22 to 1850 Mar. 29, which (with one exception) are taken with a 7-inch reflector, and which give results systematically different from those of the corresponding portions of other periods obtained with the usual instruments. Group B consists of periods II. to XI., omitting the observations of 1857 Nov. 18 to 1858 Mar. 26, which (with few exceptions) are taken with T 35, or R (7), and to which the same remark applies. Group C consists of periods XXVII. to XXXVI.

It will be seen that A and B are almost identical in regard to the position of maximum and minimum, while in C these positions are shifted about 2 "portions" (or 52 days) further on; showing that the average length of period between A and B has been about 312 days, while that between B and C has been about 2 days longer. A mean curve has been formed by shifting all the "portions" of A and B one step forward, and those of C one step back, and then taking the weighted mean. This gives a very smooth curve, with its minimum at about 133 days after the point which corresponds with the initial date of a period as given in Table XII. The magnitudes from the curve at the middle of each "portion" are given at the head of Table XII.

Owing to the position of the variable, and to the fact of its periods approaching to a year in length, no single period can be fully observed, and each "portion" is necessarily unobserved for 2 to 4 periods in succession. The apparent large differences in *general* brightness between one period and another make it especially unfortunate that both rise and fall are seldom obtained in the same period. These circumstances make calculations for "corrections to epoch" from individual periods difficult and uncertain; but they have been dealt with in what seemed the best manner possible.

TABLE XII.

Comparison of the Observations with a Mean Light Curve.

Period.	Initial Date 239+.	Number and Mean Magnitude of Portions.											
		1. 99.	2. 92.	3. 82.	4. 72.	5. 62.	6. 58.	7. 63.	8. 71.	9. 77.	10. 86.	11. 94.	12. 101.
XVIII.	3136	...	...	– 12	– 10·5	...	...	...	..	...	...	...	...
– XVI.	3760	...	...	...	...	...	...	...	+ 10	...	...	...	...
– XI.	5320	...	– 3·5	...	...	...	...	...	...	...	...	...	...
– VIII.	6256	...	...	...	– 12	– 5	+ 2	+ 5	+ 8	+ 8	+ 7	+ 13	..
– VII.	6568	...	...	..	...	+ 2	+ 10	+ 16	+ 19	+ 20	+ 10·5	+ 11·5	...
– VI.	6880	...	...	...	...	...	...	+ 14	+ 10	+ 11·5	...	+ 10	...
· V.	7192	– 6	...	...	– 3	...	...	...	...	...	+ 7	...	+ 5
– IV.	7504	– 12	– 7	– 7	– 11	+ 1	...	...	...	...	...	...	...

TABLE XII.—*Comparison of the Observations with a Mean Light Curve*—continued.

Period.	Initial Date 239+.	Number and Mean Magnitude of Portions. 1. 99.	2. 92.	3. 82.	4. 72.	5. 62.	6. 58.	7. 63.	8. 71.	9. 77.	10. 86.	11. 94.	12. 101.
– III.	7816	...	−2	...	−11	...	...	+11	...	...	...	...	...
– II.	8128	...	...	−7	...	...	+4	+10	+10	...	...	...	...
– I.	8440	...	...	...	...	...	+3	+9	+12	+8	−2	+2	...
0	8752	...	...	...	...	...	+4	+14	+9	+9	...	...	...
I.	9064	...	...	...	...	...	...	...	...	+11	...	...	...
II.	9376	−5	−5	−6	−10	−11	...	...	...	...	...	+13	+15
III.	9688	+6	+2	+2	−6	−1	+7	...	...	...	...	...	...
	(240+)												
IV.	0000	−1	−7	−3	−9·5	−6	+4	+4	+4·5	...	...	...	...
V.	0312	...	...	...	−10	−1	+6	+8	+11	+13	...	...	...
VI.	0624	...	...	...	...	...	+7	+6	+5·5	+7	+5	+5·5	+2
VII.	0936	...	...	...	...	...	...	...	+11	...	+9·5	+5	−7
VIII.	1248	−5	−8	...	...	...	...	...	...	...	+5	−1	−7
IX.	1560	−9·5	−11	−10	−9	...	...	...	...	...	+5	−2	−4
X.	1872	−13·5	−19	−17·5	−14	−6	0	...	...	...	...	...	...
XI.	2184	...	...	−9·5	−9	−3	+3	+3	+4	...	...	...	...
XII.	2496	...	...	...	−1	+1	+2	+4	+8	+12	+11	...	...
XIII.	2808	...	...	...	...	...	+1	...	+3	...	+7	...	...
XIV.	3120	...	...	...	...	...	...	...	...	...	+14	...	+10
XV.	3432	...	...	...	...	...	...	...	...	...	...	...	+4·5
XVI.	3744	−4	...	+0·5	...	...	...	...	...	...	...	...	...
XVII.	4056	+6·5	...	...	−4	...	...	...	...	...	...	...	...
XVIII.	4368	...	...	...	...	0	...	+1·5	...	...	...	...	...
XXVI.	6864	...	...	...	...	...	+1	+3	+4·5	+10	...	...	...
XXVII.	7176	...	...	...	...	+2·5	+0·5	−3	−6	...	...	−3	...
XXVIII.	7488	...	...	...	...	...	...	...	−5	−11	...	0	−5
XXIX.	7800	+3	+4	−1	...	...	...	...	...	...	−11	...	−10
XXX.	8112	+8	...	+7	+8	+11	...	...	...	...	...	...	...
XXXI.	8424	...	...	+12	+14	+12	−3	−7	...	...	...	...	...
XXXII.	8736	...	...	...	...	+10	+3	−6	...	...	...	...	
XXXIII.	9048	...	...	...	...	...	+5	+1	−4	−6·5	−11	−9·5	...
XXXIV.	9360	...	...	...	...	...	...	−6	−11	−8·5	−13	...	−9
XXXV.	9672	−4·5	...	...	...	...	...	...	...	...	−11	−10	−2·5
XXXVI.	9984	+9	+16·5	+14	...	...	...	...	...	...	...	...	−4
XXXVII.	0296	+7	+18	...	...	...	...	...	...	...	...	...	...
$p-f$.		+9	+17	+20	+20	+14	−1	−13	−14	−15	−17	−15	−5
Mean Mag. of Harvard Obs., periods XLV. to LIV. (omitting LI.).		99_3	93_4	80_4	67_2	61_2	58_4	67_1	70_5	78_3	88_4	95_1	99_2
Harvard – Curve.		0	+1	−2	−5	−1	0	+4	−1	+1	+2	+1	−2

The observations have been compared individually with the mean light curve, and the results are shown in Table XII., where the mean for each "portion" of a period is given. To avoid multiplication of figures, the numbers of observations are not given in the table, but they have been used in calculating the "corrections to epoch." In order to obtain additional evidence, with a view to deducing a formula for variation of period, the Harvard observations of this variable from J.D. 2412138 to 2415750 have been made use of. They are about equal to those of Baxendell in the matter of scantiness, and are even more discordant; but, nevertheless, the mean maximum of the "portions" in the last 10 periods (omitting one period which is specially discordant) agrees remarkably well with the Baxendell mean curve, as is shown at the foot of Table XII. The subscript figures give the number of periods in which the "portions" are represented, and their fewness makes the agreement still more remarkable. The periods have been treated in the same manner as those of S Delphini (*M.N.*, lxxiv. p. 589), except that it was not necessary to make any rearrangement such as was made in that case because of the position of maximum. The variations in *general* brightness from one period to another, and also in the apparent light curve, are such that each period has been treated separately as a rule. Further attention has been chiefly confined to such periods as show both rise and fall. For periods —VIII. and LIV. the dates of maximum have been read by inspection only. Table XIII. shows the corrections to epoch obtained by the usual process, omitting portions (6) and (12) as too near the maximum or minimum to give contributions of value. Results from specially doubtful or scanty data are marked with an *.

TABLE XIII.

Period.	No. of Periods Combined.	Correction.	Period.	No. of Periods Combined.	Correction.
– VIII.	(Insp.)	–14·0	XVIII.	1	– 1·7*
– V.	2	–24·0	XXVII.	1	+12·2*
– III.	1	–24·6*	XXIX.	2	+22·2
– I.	1	–27·9	XXXII.	1	+26·1
+I. & +II.	1	–26·4	XXXIII.	1	+31·7
III.	1	–25·2*	XXXV. & VI.	1	+34·0
IV.	3	–25·0	XLIV. & V.	1	+22·1
V.	1	–29·5	XLIX.	2	+17·7
VIII.	2	–25·8	LII.	1	+13·0*
IX.	2	–27·1	LIV.	(Insp.)	+12·0
X.	1	–24·6			
XI.	3	–23·1			
XII.	1	–23·7			

On the usual hypothesis of a cycle in the length of period, the run of these corrections suggests a curve of variation occupying for a half cycle between 26 and 36 periods, unless we make a very large correction to the adopted period. It is less easy than in

the case of S Delphini to fix the places where the cycle has its maximum and minimum, even with the help of the Harvard observations; but it seems clear that Chandler's formula

$$2362902 + 312{\cdot}9E + 25 \sin (2^{\circ}{\cdot}75E + 318^{\circ})$$

makes the cycle too long, viz. $360/2{\cdot}75 = 131$ periods, or 65 for the half cycle.

The formula which fits the observations best on the whole is

$$2401061{\cdot}7 + 312{\cdot}63E + 20{\cdot}5 \sin (6^{\circ}{\cdot}5E + 271^{\circ}),$$

which is compared with the observations in Table XIV. under the heading "Obs.—M.A.B.," a comparison with Chandler being added. The correlation formulæ (*M.N.*, lxviii. p. 551) would give a term 27 sin (8°·2 E + const.).

But better results are obtained by abandoning the idea of a cycle entirely and substituting that of two sudden changes of period. Up to period IX. a uniform period of 311·80 days suits the determinations. Then there is a sudden change to a period of 314·44 days, which leads to period XXXIII. or XXXIV.; and then there is another sudden change to 310·70 days. The residuals for this hypothesis of discontinuity are shown in the last two columns of Table XIV. under the headings "Discontinuity" and "Obs.—Dis."

TABLE XIV.

Comparison of Deduced Epochs of Maximum with Formulæ.

Period.	Observed Max. 23+.	Obs.—Chandler.	Obs.—M.A.B.	Discontinuity.	Obs.—Dis.
−VIII.	96375·0	+16·5	+0·4	96364·4	+10·6
− V.	97301·0	+ 4·7	−4·5	97299·8	+ 1·2
− III.	97924·4*	+ 2·5	−2·0	97923·4	+ 1·0*
− II.	98233·1	− 1·6	−3·9	98235·2	− 2·1
I.	99170·6	− 3·6	+1·2	99170·6	0·0
III.	99795·8*	− 4·0	+3·2	99794·2	+ 1·6*
IV.	00108·0	− 5·0	+3·6	00106·0	+ 2·0
V.	00415·5	−10·7	−0·9	00417·8	− 2·3
VIII.	01355·2	−11·1	+1·2	01353·2	+ 2·0
IX.	01665·9	−13·8	−1·2	01665·0	+ 0·9
X.	01980·4	−12·8	0·0	01979·4	+ 1·0
XI.	02293·9	−12·9	0·0	02293·9	0·0
XII.	02605·3	−15·1	−2·5	02608·3	− 3·0
XVIII.	04499·3*	− 3·6	+4·9	04495·0	+ 4·3*
XXVII.	07321·2	− 7·6	−6·5	07324·9	− 3·7*
XXIX.	07955·2	− 1·8	−1·0	07953·8	+ 1·4
XXXII.	08895·1	− 2·2	−2·0	08897·1	− 2·0*
XXXIII.	09212·7	− 0·7	+2·4	09211·5	+ 1·2
XXXV.	09839·0	− 3·3	+3·0	09840·4	− 1·4
XLV.	12947·1	−33·3	−3·0	12947·4	− 0·3
XLIX.	14190·7	−45·8	−0·1	14190·2	+ 0·5
LII.	15122·0*	−55·3	+0·1	15122·3	− 0·3
LIV.	15745·0	−59·3	+1·9	15743·7	+ 1·3

Disregarding the first determination by inspection, which is subject to considerable uncertainty, there seems no doubt that the hypothesis of discontinuity fits the observations as closely as can be expected.

The Rev. T. E. R. Phillips, in discussing the determinations of maxima of S Herculis (*Mon. Not.*, lxxv. p. 563) found it necessary to abandon the hypothesis of a simple sine curve to represent the variations of maxima; and he adopted as an alternative a formula extending to the fifth harmonic. But a glance at his diagram (plate xix., *loc. cit.*) shows that three straight lines will represent the observations very well, the changes being at about E = 13 and E = 47. The 3 uniform periods would thus be determined as follows :—

E =	1.	13.	48.	70.	
Error of Epoch =	+10	−65	+75	−35	days
Correction to Period =		−6·25	+4·00	−5·00	,,
Period		299·95	310·20	301·20	,,

The epochs of change are approximately at J.D. 2403200 and 2414000 : interval 10800 days. In the above case of R Leonis, which has nearly the same period, the dates are J.D. 2401700 and 2409800 : interval 8100 days. The hypothesis of discontinuous change seems to be worthy of attention.

The analysis of the mean curve, with the arbitrary starting point as zero, gives

$$2{\cdot}00 \cos(\theta - 336^\circ) + 0{\cdot}16 \cos 2(\theta - 21^\circ) + 0{\cdot}13 \cos 3(\theta - 88^\circ).$$

Dividing by the range (4·5 magnitudes) and multiplying by 6, to make the figures comparable with others previously published, we get

$$2{\cdot}67 \cos(\theta - 336^\circ) + 0{\cdot}21 \cos 2(\theta - 21^\circ) + 0{\cdot}18 \cos 3(\theta - 88^\circ).$$

This expression has its maximum negative value at about $\theta = 148^\circ$, or 128 days after the arbitrary starting point, which is 8 days later than the maximum as read from the curve. If this be correct, all the maxima except those found by inspection would be brought forward 8 days, the initial date of the formula would require correction, and other modifications might be necessary. On the discontinuity hypothesis very slight changes would be required. But harmonics of higher orders are liable to alter the maximum, so that the reading from the curve is perhaps to be preferred.

Determinations from different parts of the series are as follows :—

$$-\text{VIII. to } +\text{I. } 2{\cdot}62 \cos(\theta - 309^\circ) + 0{\cdot}28 \cos 2(\theta - 174^\circ) + 0{\cdot}32 \cos 3(\theta - 68^\circ)$$

$$\text{II. to XI. } 2{\cdot}78 \cos(\theta - 306^\circ) + 0{\cdot}13 \cos 2(\theta - 4^\circ) + 0{\cdot}08 \cos 3(\theta - 36^\circ)$$

$$\text{XXVII. to XXXVI. } 2{\cdot}73 \cos(\theta - 6^\circ) + 0{\cdot}28 \cos 2(\theta - 33^\circ) + 0{\cdot}11 \cos 3(\theta - 113^\circ)$$

The differences in phase of the principal term are in good agreement with the corrections to epoch.

No. 15, S Orionis (1944 *Chandler*, 052404 *Harvard*).

$5^h\ 21^m\ 51^s - 4^\circ\ 48'\cdot7$ (1855·0).
5 24 5 −4 46·4 (1900·0).

The Comparison Stars.—Baxendell gives one small sketch of these, and a chart; and the two agree together. He does not appear to have changed the designations at all. They are identified as in Table XV.

TABLE XV.

Comparison Stars.

Bax.	A.S.V.		H.A.		Adopted Mag.
	No.	Grad.	Letter.	Mag.	
a	1	...	*a*	7·62	76
c	17	...	...	...	94
b	21	...	*g*	9·76	95
d	24	...	*h*	10·25	103
e	33	...	*l*	11·42	114
f	35	...	*m*	11·60	116
g	37	...	*q*	12·14	121
h	57	...	...	...	127
k	...	...	...	...	132

The magnitudes adopted are those of Harvard (where these are given), except for the star "*b*," which Baxendell evidently saw relatively brighter as a rule.

Baxendell's differences between his comparison stars are shown in Table XVI. There is a very marked difference between those of 1871 and those of the later years, as regards nearly all the stars. A different telescope—Gl. ($7\frac{1}{4}$)—was used in the first year; E 80 being used from 1878, and Dancer eyepiece, 1878 Mar. 21. But the small intervals between his *a* and *b* and *a* and *c* continue in 1878 and 1879. He makes *b* sometimes brighter and sometimes fainter than *c*, and some others seem variable.

TABLE XVI.

Baxendell's Differences between his Comparison Stars.

Years.	94–76.	95–76.	95–94.	103–94.	103–95.	114–103.	116–114.
1871	...	$16\cdot3_7$	$-1\cdot3_3$	$11\cdot0_2$	...	$4\cdot5_2$	$0\cdot5_2$
1878–80	$17\cdot2_{36}$	$19\cdot1_{19}$	$+0\cdot9_{47}$	$10\cdot9_{10}$	$9\cdot7_{11}$	$11\cdot6_{11}$	$2\cdot2_7$
1881–4	$28\cdot0_3$	$24\cdot5_1$	$+0\cdot9_{16}$	$12\cdot6_5$	$11\cdot6_4$	...	...
1885–7	$19\cdot4_7$	$20\cdot5_1$	$+1\cdot4_{12}$	...	...	...	...
Mean (omitting 1871)	18·5	19·4	1·0	11·5	10·2	11·6	2·2
No. of Observations	46	21	75	15	15	11	7
Reduced Mean	...	...	1·0	9·3	8·1	10·4	2·0

The "reduced mean" is obtained in the usual manner; but as Baxendell does not appear to have exaggerated the interval between *a* and *b* or *a* and *c*, except very occasionally, the scale of reduction has not been applied in the comparisons with those stars. There are three instances of 121–116,—1·0 (in 1871) and 3·0 twice (in 1878); two of 127–121, both 8·0 (reduced value 6·5); and two of 132–127—one (in 1871) 1·0, and the other (in 1878) 6·5 (reduced, 5·5).

TABLE XVII.

Baxendell's Observations of S Orionis.

Date.	J.D. 240+.	Corrected Comparisons.		Deduced Mag.
1871				
Mar. 3	4490	76+11,	95−5	88
13	4500	76+8,	95−8	86
16	4503	76+7,	95−8	85
18	4505	76+8,	95−7	86
Apr. 7	4525	76+7,	95−12	83
8	4526	76+6,	95−9	84
10	4528	76+8,	95−10	84
1878				
Jan. 10	6995	76+8,	94−5	86
25	7010	76+7,	94−8	84
28	7013	76+5,	94−7	84
31	7016	76+7,	94−10	84
Feb. 7	7023	76+5,	94−10	82
15	7031	76+7,	94−8	84
Mar. 2	7046	76+12,	95−5	89
13	7057	76+12,	94−4	89
21	7065	76+20,	94−2	94
Apr. 4	7079	95+2,	103−3	98
Nov. 4	7293	103+9,	114−2	112
25	7314	103+5,	114−1	110
28	7317	103+7,	114−1	112
Dec. 5	7324	103+6,	114−6	108
22	7341	103+0,	114−8	104
23	7342	103+1,	114−8	105
30	7349	103+1,	114−11	104
1879				
Jan. 13	7363	103−4		99
15	7365	95+3,	103−6	98
16	7366	94+2,	95+1, 103−7	96
19	7369	94+0,	95−1	94
22	7372	94+0,	95−1	94
25	7375	94−1,	95−1	94
Feb. 8	7389	76+12,	94−6	88
16	7397	76+3,	94−10	82
17	7398	76+4,	94−10	82
21	7402	76+7,	94−8	84
25	7406	76+3,	94−12	80
28	7409	76+2,	94−14	79
Mar. 1	7410	76+0,	94−14	78
3	7412	76+1,	94−14	78
6	7415	76+0,	94−15	78
8	7417	76+2,	94−14	79
29	7438	76+2,	94−14	79
31	7440	76+3,	94−12	80
Apr. 3	7443	76+3,	95−12	81
5	7445	76+6,	94−12	82
10	7450	76+8,	94−8	85
Sept. 28	7621	116+0?		*116?
Oct. 14	7637	116−1?		*115?
15	7638	116−1?		*115?
20	7643	116−1?		*115?
Nov. 1	7655	116−1?		*115?
2	7656	116−1?		*115?
12	7666	116+0?		*116?
22	7676	114+0		114
25	7679	103+8,	114−4	110

* "Have I mistaken *e* for S in my late observations?"—Baxendell, Nov. 12.

TABLE XVII.—*Baxendell's Observations of S Orionis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1879			
Nov. 27	7681	103+4, 114−7	107
28	7682	103+6	109
29	7683	103+5, "strong ☽"	108
30	7684	103+6 ,,	109
Dec. 3	7687	103+6, 114−7, "hazy"	108
4	7688	103+3, 114−8	*106
23	7707	103−2	101
24	7708	103−4, "hazy and ☽"	99
1880			
Jan. 2	7717	94+1, 103−5	96
18	7733	103−9, 95−1, 76+14, 94−2	92
19	7734	76+14, 103−11, 95−3, 94−3	91
22	7737	76+17, 95−3, 94−3	92
28	7743	76+14, 95−4, 94−5	90
29	7744	76+14, 95−5, 94−6	89
31	7746	76+13, 95−6, 94−7	88
Feb. 1	7747	95−6, 94−7	88
4	7750	76+16, 95−7, 94−8	89
8	7754	76+14, 95−8, 94−8	88
10	7756	76+14, 95−6, 94−6	89
20	7766	76+13, 95−10, 94−11	86
Mar. 7	7782	76+12, 95−8, 94−8	87
17	7792	76+13, 95−13, 94−14	84
1880			
Apr. 2	7808	76+8, 95−10, 94−12	84
8	7814	76+5, 95−16	80
Oct. 10	7999	132−2	130
Nov. 8	8028	More than 116+6	<122
Dec. 1	8051	132−3	129
20	8070	114+2, 121−2	118
24	8074	114−1	113
25	8075	114−1	113
30	8080	114−3	111
1881			
Jan. 12	8093	103+0	103
Feb. 20	8132	103+0	103
23	8135	103+0	103
28	8140	103+0	103
Mar. 11	8151	103−4	99
28	8168	103−4, 95+7, 94+6	101
31	8171	94+8, 103−3, 95+7	101
Oct. 29	8383	"12·7 to 13·0 mag., but very low"	
Nov. 17	8402	"About 12·8."	
Dec. 10	8425	"Not visible, <12·2"	
19	8434	"About 12·8 or 13."	
27	8442	121+5	126
1882			
Feb. 12	8489	114+0	114
Mar. 6	8511	103+3	106
12	8517	103+0	103
20	8525	103+0	103
Apr. 7	8543	103+1	104
1883			
Jan. 30	8841	"Seen with difficulty."	

* "Better observation than any lately."—Baxendell.

TABLE XVII.—*Baxendell's Observations of S Orionis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1883			
Feb. 22	8864	<12·5, ☽ ☽	
Mar. 5	8875	132 − 5	127
12	8882	121 + 7	128
26	8896	121 + 0	121
Apr. 1	8902	"Invis. ; *f* seen."	
1884			
Jan. 20	9196	127 + 0	127
Feb. 2	9209	127 + 0	127
11	9218	Invis. <116.	
14	9221	116 + 4, "hasty observation"	120
25	9232	132 − 4	128
Mar. 16	9252	116 + 9	125
23	9259	116 + 9	125
Nov. 2	9483	76 + 15, 94 − 17	84
9	9490	76 + 17, 94 − 10	88
14	9495	76 + 17, 94 − 7, 95 − 7	89
20	9501	94 − 5	89
24	9505	94 − 1, 95 − 2	93
Dec. 10	9521	94 + 7, 103 − 3, 95 + 6	101
15	9526	94 + 3, 103 − 6, 95 + 2	97
20	9531	94 + 9, 103 − 3, 95 + 8	102
25	9536	103 − 1	102
1885			
Jan. 5	9547	103 + 2	105
Feb. 2	9575	114 + 3	117
Nov. 15	9861	94 − 6	88
17	9863	94 − 7	87
Dec. 1	9877	76 + 14, 94 − 6, 95 − 7	89
26	9902	94 − 7, 95 − 8	87
1886			
Jan. 2	9909	94 − 3, 95 − 4	91
5	9912	94 − 4, 95 − 5	90
30	9937	94 + 2, 95 − 1	95
Feb. 14	9952	94 + 3, 95 + 2	97
24	9962	103 − 1	102
Mar. 3	9969	103 − 1	102
	241 +		
Nov. 17	0228	94 − 4, 95 − 5	90
30	0241	94 − 7, 95 − 8	87
Dec. 29	0270	94 − 3, 95 − 4	91
1887			
Jan. 12	0284	94 − 9, 95 − 10	85
26	0298	76 + 5, 94 − 7	84
29	0301	76 + 5, 94 − 12	82
Feb. 10	0313	76 + 8, 94 − 13	82
14	0317	76 + 7, 94 − 12	82
18	0321	76 + 10, 94 − 15	82
27	0330	76 + 12, 94 − 10	86

Chandler's formula is 2404095 + 413E, "large irregularity."

A period of 414 days was assumed, and the observations were tabulated in the usual manner, and the means taken for groups of periods, with the following result:—

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
A . [I.]	...	...	87	84	...	...	...	...	...	...	...	...
B [VII. + (4)].	100	95	87	81	83	94	...	...	130	128	115	106
C [XII. + (3)].	90	87	87·5	87	99	103	117	127	126	...	...	...

A mean light curve has been formed by shifting C one "portion" forward, and then taking the mean of the figures, thus obtaining:—

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Mean Curve	100	93	87	84	85	96	103	117	128	127	115	106

This makes not at all a bad curve, but it is too uncertain, and the observations are too few for it to be worth while to treat it in the usual manner.

The corrections to epoch can be estimated somewhat as follows, adopting Chandler's formula above:—

Period.	I.	VII.	VIII.	IX.	XIII.	XIV.	XV.
Obs.—Chand.	$+32^d$	$+35^d$	$+17^d$	$>+2^d$	$<+19^d$	$+13^d$	$+16^d$
	or $+19$?					or $+6^d$	

Periods I. and XIV. are susceptible of two different interpretations. It is clear that no very definite correction to period can be made from such material.

The analysis of the above mean light curve gives, when divided by the range and multiplied by 6:

$$2{\cdot}70\cos(\theta - 260^\circ) + 0{\cdot}48\cos 2(\theta - 58^\circ) + 0{\cdot}28\cos 3(\theta - 16^\circ).$$

Comparison of the Bordeaux, Washington, and Algiers Catalogues.
By R. J. Pocock, B.A., B.Sc.

Of the stars in the Bordeaux Catalogue of 6999 stars between -15° and -20°, over 4000 are contained in the Washington A.G. Catalogue; a detailed comparison of these two catalogues has been carried out in connection with the Hyderabad astrographic work. Omitting stars with large proper motions, etc., 4200 stars have been compared, and the results are summarised in the present note.

The Washington and Algiers A.G. Catalogues overlap, and both are required in determining the plate constants for the Hyderabad astrographic plates. The Algiers Catalogue is not yet published, but a type-written copy has been placed at our disposal by the Director of the Paris Observatory, and this copy includes a comparison of the stars common to Washington and Algiers. These differences also are summarised in this note. Since the epoch of the Bordeaux Catalogue is 1890·0 and of the other two catalogues 1900·0, the Bordeaux places had to be brought up to 1900·0 by means of the precessions and secular variations given in the Bordeaux Catalogue, which depend on Le Verrier's constants.

The results Bord.–Alg. have not been obtained by direct comparison, but are differences between the results Bord.–Wash. and Alg.–Wash. The number of stars used in the comparison Alg.–Wash. is 639.

In Table I. are given the means of the differences in R.A. and Decln. for each hour of R.A.

TABLE I.

R.A.	Bord. – Wash.		Alg. – Wash.		Bord. – Alg.	
	s	″	s	″	s	″
0	– ·021	+0·12	+ ·044	– 0·39	– ·065	+0·51
1	– ·009	+0·67	+ ·006	– 0·28	– ·015	+0·95
2	+ ·011	+1·02	– ·025	– 0·69	+ ·036	+1·71
3	+ ·018	+1·03	– ·021	– 0·36	+ ·039	+1·39
4	+ ·058	+1·05	+ ·012	– 0·33	+ ·046	+1·38
5	+ ·064	+0·80	+ ·062	– 0·50	+ ·002	+1·30
6	+ ·046	+0·82	+ ·056	– 0·36	– ·010	+1·18
7	+ ·046	+0·70	+ ·066	– 0·05	– ·020	+0·75
8	+ ·046	+0·56	– ·035	– 0·34	+ ·081	+0·90
9	+ ·030	+0·35	+ ·034	– 0·43	– ·004	+0·78
10	+ ·036	+0·31	+ ·026	– 0·41	+ ·010	+0·72
11	+ ·030	+0·40	+ ·049	– 0·31	– ·019	+0·71
12	+ ·027	+0·38	+ ·028	+0·12	– ·001	+0·50
13	+ ·009	+0·15	+ ·042	– 0·51	– ·033	+0·66
14	– ·007	– 0·09	– ·010	– 0·36	+ ·003	+0·27
15	– ·021	– 0·28	– ·056	– 0·06	+ ·035	– 0·22
16	– ·034	– 0·57	+ ·016	– 0·36	– ·050	– 0·21
17	– ·018	– 0·73	+ ·041	0·00	– ·059	– 0·73
18	– ·022	– 0·42	+ ·014	– 0·04	– ·036	– 0·38
19	– ·016	– 0·40	+ ·009	+0·09	– ·025	– 0·31
20	– ·037	– 0·42	+ ·042	– 0·24	– ·079	– 0·18
21	– ·020	– 0·43	+ ·014	+0·07	– ·034	– 0·50
22	– ·033	– 0·29	+ ·017	– 0·83	– ·050	+0·54
23	– ·053	+0·10	– ·001	– 0·40	– ·052	+0·50
Means	+ ·005	+0·20	+ ·018	– 0·29	– ·013	+0·51

The differences in R.A. Bord.–Wash. may be divided into two twelve-hour groups. From 2^h to 13^h they are practically constant equal to $+^s·035$, though there is some evidence of curvature which may be represented approximately by the formula

$$46 - \frac{5}{4}(7 - \alpha)^2$$

where α expresses the R.A. in hours, and the unit is $^s·001$. From 14^h to 1^h they are approximately represented by the constant $-^s·024$.

The differences in R.A. Alg.–Wash. show evidence of a six-hour group, as is seen in Table II.

TABLE II.

Differences in R.A. Alg.–Wash. in six-hour groups.
(*unit* ^{s}·001.)

	+ 44	+ 6	− 25	− 21	+ 12	+ 62
	+ 56	+ 66	− 35	+ 34	+ 26	+ 49
	+ 28	+ 42	− 10	− 56	+ 16	+ 41
	+ 14	+ 9	+ 42	+ 14	+ 17	− 1
Sum	+ 142	+ 123	− 28	− 29	+ 71	+ 151

The differences in R.A. Bord.–Alg. also show some evidence of a six-hour group, though not so strongly, due to the Algiers Catalogue.

The differences in Decln. Bord.–Wash. may be represented by the formula

$$0\cdot20 + 0\cdot723 \sin (15a + 6)^{\circ}.$$

If we omit the constant and group together similar terms (changing the sign where necessary), so as to smooth the results, we get:

Obsd.	+ ·01	+ ·28	+ ·47	+ ·57	+ ·68	+ ·70
Calcd.	+ ·09	+ ·28	+ ·44	+ ·56	+ ·68	+ ·72

The first half of the differences in Decln. Alg.–Wash. from 0^{h} to 11^{h} are practically constant and equal to $+''\cdot37$. The mean from 12^{h} to 23^{h} is $+''\cdot21$, but after subtracting this the differences are still large and irregular.

The differences in Decln. Bord.–Alg. may be represented by the formula

$$0\cdot51 + 0\cdot813 \sin (15a + 16)^{\circ}.$$

Smoothing the numbers in the same way as before, we obtain

Obsd.	+ ·06	+ ·20	+ ·68	+ ·67	+ ·66	+ ·90
Calcd.	+ ·10	+ ·30	+ ·50	+ ·64	+ ·74	+ ·80

To test the variation of the differences with the magnitude, the first ten stars in each hour of R.A. were used. The results are exhibited in Table III.

TABLE III.

Magnitude.	Bord. – Wash.		Alg. – Wash.		Bord. – Alg.	
	^{s}	″	^{s}	″	^{s}	″
7·5 and brighter	$-\cdot007_{15}$	− ·65	$+\cdot013_{18}$	− ·05	− ·020	− ·60
7·6 to 8·0	$+\cdot015_{22}$	− ·34	$+\cdot032_{17}$	− ·11	− ·017	− ·23
8·1 to 8·5	$+\cdot028_{24}$	− ·28	$+\cdot023_{42}$	− ·28	+ ·005	·00
8·6, 8·7	$+\cdot004_{31}$	+ ·58	$+\cdot045_{34}$	− ·16	− ·041	+ ·74
8·8, 8·9	$+\cdot032_{36}$	+ ·17	$+\cdot016_{60}$	− ·36	+ ·016	+ ·53
9·0	$-\cdot043_{49}$	+ ·35	$+\cdot007_{26}$	− ·09	− ·050	+ ·44
9·1	$-\cdot028_{19}$	+ ·48	$+\cdot017_{20}$	− ·37	− ·045	+ ·85
9·2	$+\cdot012_{24}$	+ ·27	$-\cdot003_{15}$	+ ·04	+ ·015	+ ·23
9·3 and fainter	$-\cdot006_{28}$	+ ·41	$-\cdot028_{8}$	+ ·20	+ ·022	+ ·21

The suffixes denote the number of stars in each group. As before, the last two columns are obtained from the four preceding columns, not by direct comparison between the two catalogues.

If we take the mean of the numbers in Table III. down to 8·9, and the mean of the rest, we get,

	Bord. – Wash.		Alg. – Wash.		Bord. – Alg.	
	s	″	s	″	s	″
Bright stars	+ ·030	– ·48	+ ·028	– ·13	+ ·003	– ·34
– faint						

Nizamiah Observatory,
Hyderabad:
1915 *August* 20.